Infinite Series
Class 25: Wednesday April 2

Example 1 $\sum_{k=1}^{\infty} \frac{1}{k}$ (This is called the **HARMONIC SERIES**.)

Partial sums (fill in the sums):

$S_1 = 1 =$

$S_2 = 1 + 1/2 =$

$S_3 = 1 + 1/2 + 1/3 =$

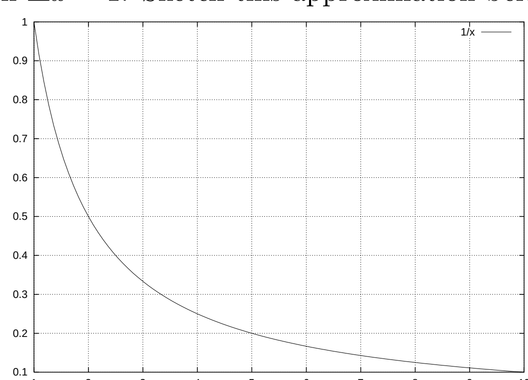
$S_4 = 1 + 1/2 + 1/3 + 1/4 =$

$S_5 = 1 + 1/2 + 1/3 + 1/4 + 1/5 =$

Do you think these partial sums have a limit?

We need to come up with a systematic way of determining the convergence or divergence of an infinite series. Over the next week or so we will learn about **Convergence Tests**.

Let us look at the Left-hand Riemann Sum approximation **L** of the area under the curve $f(x) = 1/x$ from $a = 1$ up to $b = 10$ with $\Delta x = 1$. Sketch this approximation below...



Is **L** an over-estimate or an under-estimate?

What is the relationship between the Left-hand Riemann Sum $LEFT(10)$, S_{10} and the $\int_1^{10} \frac{1}{x}\,dx$? Write in those relationships (<, > =, etc) below...

$LEFT(10)$ $\qquad\qquad$ S_{10} $\qquad\qquad$ $\int_1^{10} \frac{1}{x}\,dx$

What happens if instead of 10 we sum up to 1000? 100000? Infinity?

So, by geometry we can show that $\sum_{i=1}^{\infty} \frac{1}{k}$, the HARMONIC SERIES, ______________________

1. INTEGRAL TEST If $a(k) > 0$ for all k If $\int_1^\infty a(k)\,dk$ CONVERGES, then $\sum_{k=1}^\infty a(k)$ CONVERGES.

If $\int_1^\infty a(k)\,dk$ DIVERGES, then $\sum_{k=1}^\infty a(k)$ DIVERGES.

GroupWork

Determine whether the following infinite series CONVERGE or DIVERGE.

Example 2 $\sum_{k=1}^\infty \frac{1}{k^2}$

Example 3 $\sum_{k=1}^\infty k^2$

Example 4 $\sum_{k=1}^\infty \frac{1}{\sqrt{k}}$

Connection Between Improper Integrals of the First Kind and Infinite Series

By applying the integral test to the infinite series $\sum_{k=1}^\infty \frac{1}{k^p}$ and reviewing the examples above fill in the appropriate condition on p in the RULE below

$$\sum_{k=1}^\infty \frac{1}{k^p} \begin{cases} \textbf{CONVERGES} & \text{when } p \\ \textbf{DIVERGES} & \text{when } p \end{cases}$$