Introduction to Infinite Series
Class 24: Monday March 31

Summing an Infinite List of Numbers.
If someone gives you a list of numbers, even a long list of numbers (like 1000 of them), it is at least theoretically possible for you to use your calculator or computer to find the sum total of this list. Now, suppose someone gives you an *infinite* list of numbers, for example:

$$1, \quad 1/2^2, \quad 1/3^3, \quad 1/4^4, \quad 1/5^5, \quad 1/6^6, \quad \textit{etc.}$$

Is it possible to find the total? What could "find the total" mean if you are adding up an infinite list of numbers?

GroupWork In small groups use your calculators to begin with the first number on the infinite list above, 1, and progressively add each successive number on the list, keeping track of the subtotals you get by placing them in the chart below, with seven places after the decimal.

n	n^{th}	subtotal
1	1	$= 1.0000000\ldots$
2	$1 + 1/2^2$	$=$
3	3^{rd} subtotal	$=$
4	4^{th} subtotal	$=$
5	5^{th} subtotal	$=$
6	6^{th} subtotal	$=$

What do you find happening to the subtotals? If this trend continues, what will be the first four digits of all the subtotals beyond those in the table? None of the numbers in the list, beyond a certain point, seem to be affecting the first four digits of the subtotals. So, if you were somehow able to add up *all* of the numbers in the infinite list, what do you think the first four digits of the total would be?

Find the first six decimals of the sum of the numbers in our infinite list.

What would you do to find the first ten decimals of the sum of the numbers in ou r infinite list? (You don't have to actually do it.)

How would you describe the sum of our infinite list of numbers using the concept of "limit"?

Formal Language of Infinite Series.

Using the proper terminology, we will discuss what you have just done. We had a list of numbers (which is called a *sequence* of numbers):

$$1, \quad 1/2^2, \quad 1/3^3, \quad 1/4^4, \quad 1/5^5, \quad 1/6^6, \quad \text{etc.}$$

which we call the **TERMS** of the **INFINITE SERIES**

$$1 + 1/2^2 + 1/3^3 + 1/4^4 + 1/5^5 + 1/6^6 + \cdots = \sum_{k=1}^{\infty} 1/k^k.$$

We tried to find the sum of this infinite series by looking at its **SEQUENCE OF PARTIAL SUMS** (list of subtotals):

$$\begin{aligned} S_1 &= 1 \\ S_2 &= 1 + 1/2^2 \\ S_3 &= 1 + 1/2^2 + 1/3^3 \\ S_4 &= 1 + 1/2^2 + 1/3^3 + 1/4^4 \\ &\vdots \\ S_n &= 1 + 1/2^2 + 1/3^3 + 1/4^4 + \cdots + 1/n^n \\ &\vdots \end{aligned}$$

We found that the sequence of partial sums S_n seemed to have a **LIMIT** (the subtotals were stabilizing), and that the limit of this sequence of partial sums was the **SUM** of the infinite series:

$$\sum_{k=1}^{\infty} 1/k^k = \lim_{n\to\infty} S_n.$$

When the partial sums S_n of an infinite series have a limit, the infinite ser ies is said to **CONVERGE**. When the partial sums S_n do not have a limit, the infinite series is said to **DIVERGE**. Therefore, in this case, the infinite series that we have been examining converges.

Some Examples.

Example 1 $\quad \displaystyle\sum_{k=1}^{\infty} (-1)^k$

Partial sums (fill in the sums):

$S_1 = (-1) =$

$S_2 = (-1) + 1 =$

$S_3 = (-1) + 1 + (-1) =$

$S_4 = (-1) + 1 + (-1) + 1 =$

From your partial sums above, do the partial sums have a limit?

Does the infinite series converge?