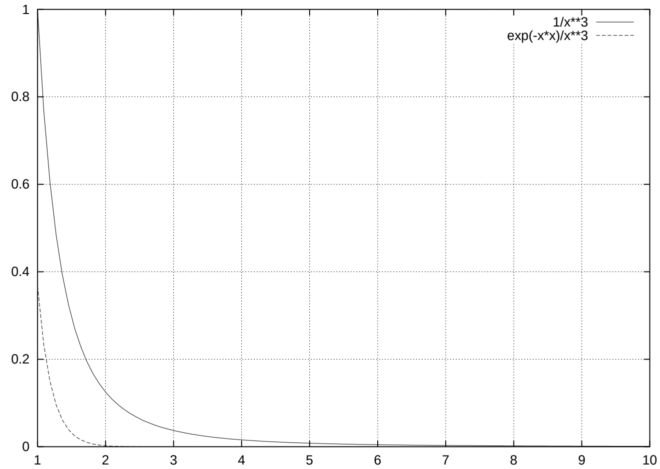
Comparing Improper Integrals
Class 23: Friday March 28

1. Consider the graphs of $f(x) = \dfrac{e^{-x^2}}{x^3}$ and $g(x) = \dfrac{1}{x^3}$ for $x \geq 1$ and sketch them below:



(a.) Is the area under $f(x)$ less than or greater than the area under $g(x)$?

2. Does $\displaystyle\int_1^\infty \frac{1}{x^3}dx$ converge or diverge? Why?

(a.) Therefore, what can you say about $\displaystyle\int_1^\infty \frac{e^{-x^2}}{x^3}dx$? Does it converge or diverge? Why?

Comparison Test for Improper Integrals

1. If $g(x) > f(x) > 0$ for all $x > a$ then if $\int_a^\infty g(x)\ dx$ CONVERGES, then $\int_a^\infty f(x)\ dx$ also CONVERGES.
2. If $f(x) > g(x) > 0$ for all $x > a$ then if $\int_a^\infty g(x)dx$ DIVERGES, then $\int_a^\infty f(x)dx$ also DIVERGES.

NOTE: You always compare the function and integral you're not sure about ($f(x)$) to the function you DO know about ($g(x)$).
Also you need to decide whether you are trying to prove convergence or divergence FIRST before you pick a function to compare to.

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3. Consider $\mathcal{I} = \int_1^\infty \frac{1}{\sqrt{x^3+5}}dx$ and compare it to the integral $\int_1^\infty \frac{1}{x^{3/2}}dx$.
Does $\mathcal{I}$ converge or diverge? Why?

4. Consider $\mathcal{K} = \int_1^\infty e^{t^2+t+1}\,dt$. Do you think this integral converges or diverges? PROVE IT!

BONUS HOMEWORK: (5 points: Due Fri 4/3/03)
Consider the following integrals and, using the Comparison Test for Improper Integrals, determine whether they converge or diverge. Make sure you state clearly what integral you are choosing to compare the given integral to, and how you know your chosen integral converges or diverges.

5. $\displaystyle\int_1^\infty \frac{1}{\sqrt{s^2+s}}ds$

6. $\displaystyle\int_1^\infty e^{-x^4}dx$

7. $\displaystyle\int_0^\pi \frac{2-\sin w}{w^2}dw$

8. $\displaystyle\int_0^\pi \frac{\sin^2(x)}{\sqrt{x}}dx$