More Antiderivatives and the Indefinite Integral
Class 12: Monday February 24

Integrals Practice

1. $\int_{-1}^{0} e^{2x}\ dx =$

2. $\int_{\frac{-1}{2}}^{\frac{1}{2}} (x-1)^4\ dx =$

3. $\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos^2(x)} dx =$

4. $\int_{1}^{2} \frac{3}{x^4} + \frac{x^4}{3}\ dx =$

5. $\int_{0}^{16} (4x)^{1/4}\ dx =$

6. $\int_{\pi}^{0} \cos(2x) + \pi dx =$

7. $\int_{0}^{8} \sqrt[3]{x}\ dx =$

8. $\int_{-1}^{1/2} 5x^3 + 2x^2 + 4\ dx + \int_{1/2}^{1} 5x^3 + 2x^2 + 4\ dx =$

Elementary Functions
Any function made up of "combinations" of the "familiar" functions is called an **elementary function**.

More precisely: A function is called elementary if it is obtained from x, $\sin(x)$, $\cos(x)$, e^x, $\ln(x)$, by addition, subtraction, multiplication, division, exponentiation (powers), or composition.

Example $5x^4 + \sin(e^{3x+1})$ is an elementary function.
Another example of an elementary function is _____

Surprising fact: Many functions do not have antiderivatives that can be written as elementary functions, but they nevertheless do have antiderivatives!

Example $f(x) = \dfrac{\sin(x)}{x}$ does have an antiderivative, but we just can't write it down "explicitly" (i.e., as an elementary function).

How do we know $\dfrac{\sin(x)}{x}$ has an antiderivative?

Write down the function $F(x)$ whose derivative equals $\dfrac{\sin(x)}{x}$

$$F(x) =$$

GroupWork Find the following derivatives.

1. $\dfrac{d}{dx}[(x^3 + 5x)^{14}] =$

2. $\dfrac{d}{dx}[(\sin(x))^{24}] =$

3. $\dfrac{d}{dx}[(u(x))^{32}] =$

An indefinite integral is denoted as $\int f(x)dx$ and represents the set of functions whose derivative equals $f(x)$. It is a family of functions.

Think-Pair-Share
How is a **definite integral** DIFFERENT from an indefinite integral? (write down as many differences as you can and then share your list with your nearest neighbor)

GroupWork
Find the following indefinite integrals (explicit form).

1. $\int (3x^2+5)(x^3+5x)^{13}\,dx =$

2. $\int \cos(x)[\sin(x)]^{23}\,dx =$

3. $\int u'(x)[u(x)]^{31}\,dx =$

4. $\int x^2(x^3+1)^{55}\,dx =$