Fundamental Theorem of Calculus
Class 9: Monday February 10

Definition: $f'(x)$ is the derivative of $f(x)$; so we can also say that $f(x)$ is an *antiderivative* of $f'(x)$.

Example: $3x^2$ is ________________________ of x^3, so _____ is an antiderivative of _____.
$x^3 + 7$ too is an antiderivative of _____, because ________________________.

1. Give two more functions that are antiderivatives of $3x^2$.
2. How many antiderivatives does $3x^2$ have?
3. Write down all of them!

THE FUNDAMENTAL THEOREM OF CALCULUS (PART ONE)

Theorem: For any continuous function $g(x)$, to evaluate

$$\int_a^b g(x)\,dx$$

find a function $G(x)$ that is an antiderivative of $g(x)$; then

$$\int_a^b g(x)\,dx = G(b) - G(a)$$

For more information you can read Section 4.5 of Smith & Minton.

Examples:

4. $\displaystyle\int_4^{100} \cos(x)\,dx =$

5. Find the area under the curve $f(x) = e^x$ on the interval $[-2, 2]$.

6. $\displaystyle\int_{-3}^{1} x^4\,dx =$

The Fundamental Theorem of Calculus (Part Two)

Theorem: If $f(x)$ is a continuous function on $[a, b]$ and $F(x)$ is an accunulation function for $f(x)$ defined as

$$F(x) = \int_a^x f(t)\, dt$$

then $F'(x) = f(x)$. In other words, $\frac{d}{dx}\int_a^x f(t))\, dt = f(x)$

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7. $F(x) = \int_0^x e^{-s}\, ds$. What is $F'(x)$?

8. $G(x) = \int_x^2 \ln(t^2+1)\, dt$. What is $G'(x)$?

9. $H(x) = \int_1^{x^2} e^t\, dt$. What is $H'(x)$?

10. Evaluate $\frac{d}{dx}\int_x^{\sin(x)} \sqrt{k^2+1}\, dk$