Properties of the Definite Integral
Class 8: Friday February 7

Definition of the Definite Integral

$$\lim_{N\to\infty} \sum_{k=1}^{N} f(x_k)\Delta x_k = \int_a^b f(x)\,dx$$

when $f(x)$ is some function defined on the interval $[a, b]$ split into N subintervals consisting of the slices Δx_k, and the limit $N \to \infty$ exists.

PROPERTIES OF THE DEFINITE INTEGRAL.

$$\begin{aligned}
\int_a^b [f(x) + g(x)]\ dx &= \int_a^b f(x)\ dx + \int_a^b g(x)\ dx \\
\int_a^b [f(x) - g(x)]\ dx &= \int_a^b f(x)\ dx - \int_a^b g(x)\ dx \\
\int_a^b c \cdot f(x)\ dx &= c\int_a^b f(x)\ dx \\
\int_a^c f(x)\ dx + \int_c^b f(x)\ dx &= \int_a^b f(x)\ dx \\
\int_a^b f(x)\ dx &= -\int_b^a f(x)\ dx
\end{aligned}$$

If $f(x) \le g(x)$, for all $x \in [a, b]$, then

$$\int_a^b f(x)\ dx \le \int_a^b g(x)\ dx$$

Problem A. What can you say about the relationship between $\int_1^4 x\ dx$ and $\int_1^4 \ln(x)\ dx$?

Problem B. Suppose you know that $\int_2^5 f(x)dx = -6$, $\int_2^5 g(x)dx = 9$ and $\int_{-2}^2 f(x)dx = 20$

Try to use the properties of definite integrals to evaluate the following:

1. $\int_2^5 [f(x) + g(x)]\ dx =$

2. $\int_2^5 4g(x)]\ dx =$

3. $\int_2^5 f(x) \cdot g(x)\ dx =$

4. $\int_{-2}^5 f(x)\ dx =$

5. $\int_5^2 f(x)\ dx =$

6. $\int_5^2 f(x)^2\ dx =$