The Pyramid: Estimating Volumes
Class 2: **Friday January 24**

This first part of today's class deals with the size, shape and volume of a pyramid with a square base of size 5 in by 5 in and height 12 in. (The height is measured from the apex to the pyramid to the point directly below it on the base.)

Note: There is a "known" formula for the volume of such a figure. Our purpose is to give reasons why this formula is correct. It's ok to refer to such a formula, but it is not required, and will distract from the goal of this exercise.

1. What is the area of the base of the pyramid?

2. What is the area of the square which is the cross-section of the pyramid halfway from the apex to the base?

3. Sketch a diagram of the pyramid in perspective. (Some find it easiest to start by drawing a parallelogram which represents the perspective view of the base.)

4. Draw in the cross-section of the pyramid (parallel to the base) which is x units down from the apex. Determine the area $A(x)$ of this square cross-section as a formula using the variable x.

5. Imagine the pyramid enclosed by a box with its base the same as the base of the pyramid. What is the *volume* of this outside box?

6. Imagine a box *inside* the pyramid with its bottom on the base and its top the cross-section you sketched above. What is the volume of this inside box?

7. Using the work above, estimate (roughly) the volume of the pyramid. Is your estimate an **over-estimate** or an **under-estimate**? How confident of your estimate are you?

8. To improve the estimate the volume of the pyramid, use two boxes to overestimate the volume. For simplicity's sake, use two boxes each of height 6 in.

9. Now give the 4-box overestimate. It will be very useful to organize your computations into a table.

10. To get an underestimate, remove the largest (bottom) box and move the others downward.

11. Using 6 boxes, give the overestimate and the corresponding underestimate. You should be able to do this by extrapolating from the estimates which use 1, 2 and 4-boxes

12. Can you write down a formula which gives the volume overestimate for any number (say N) of boxes?

Homework Assignment #2
(*4 points* due Monday January 27)

1. Suppose the exact formula for overestimate of the volume is $\dfrac{12}{N} 5^2 \dfrac{(1^2 + 2^2 + 3^2 + \cdots + N^2)}{N^2}$.
   You (with a partner or group) should use a computer (TI-83 or a PC with Microsoft Excel) to produce overestimates of the volume using 10 boxes, 50 boxes and 100 boxes.

2. Now reconsider the pyramid with square base of 5 cm and height 12 cm. What do your answers in the previous question suggest that the exact volume of the pyramid is?

3. Now consider the pyramid with square base of 7 cm and height 9 cm. What is its volume? What is your method?

4. (BONUS: 1 point) Suppose, with the original dimensions, that the apex of the pyramid is above one of the base's corners. How does this affect the calculation of estimates for the volume?