Class 31: Friday April 20

Alternating Series Test and Absolute Comparison Test

---

1. What do the following series have in common?

$1 - 2 + 3 - 4 + 5 - 6 + \cdots \qquad 1/1 - 1/2 + 1/3 - 1/4 + \cdots \qquad -1/1^2 + 1/2^2 - 1/3^2 + 1/4^2 - \cdots$

Series of this type are called ________________.

2. Which of the following series is alternating?

(a) $\displaystyle\sum_{k=4}^{\infty}(-1)^k\frac{3}{k}$

(b) $\displaystyle\sum_{k=4}^{\infty}(-1)^{2k}\frac{3}{k}$

(c) $\displaystyle\sum_{k=4}^{\infty}(-1)^{k+1}\frac{3}{\ln(k)}$

(d) $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{k}{1000}$

(e) $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{k+10}{3k+1}$

Theorem

If in an alternating series, $a_1 - a_2 + a_3 - a_4 + \cdots$, both of the following conditions hold, then the series converges.

1. Each term has smaller magnitude than the previous one, i.e., $a_{k+1} < a_k$.
2. $\lim_{k\to\infty} a_k = 0$.

---

3. For each of the series above that is alternating, determine which, if any, of the two conditions of the theorem hold. Which series converge, and which diverge?

Absolute Ratio Test

For any infinite series $\sum_{k=1}^{\infty} a_k$, if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$ then $\sum_{k=1}^{\infty} a_k$ CONVERGES.

If $L > 1$ or if $|a_{k+1}/a_k|$ does not exist, then $\sum_{k=1}^{\infty} a_k$ DIVERGES

If $L = 1$ the test is INCONCLUSIVE.

GroupWork

Let's use the absolute ratio test on the following series to tests for convergence

$$\sum_{k=0}^{\infty} (-1)^k \frac{k+10}{3k+1}$$

$$\sum_{k=0}^{\infty} \frac{k}{3k^2+1}$$

$$\sum_{k=0}^{\infty} \frac{k+1}{2^k}$$