Class 27: Friday April 6

Rules for Improper Integrals

GROUPWORK

Evaluate the following integrals. If the integral is improper, say what KIND of improper integral it is and determine whether it CONVERGES or DIVERGES. Look for patterns, and try and discover the rules found on the last page.

1. $\displaystyle\int_2^\infty \frac{1}{x^{1.000001}}\,dx$

2. $\displaystyle\int_5^\infty \frac{1}{s^{0.99999999}}\,ds$

3. $\displaystyle\int_0^2 \frac{1}{t^4}\,dt$

4. $\displaystyle\int_1^5 \frac{1}{t-2}\,dt$

5. $\displaystyle\int_0^8 \frac{1}{\sqrt[3]{r}}\,dr$

6. $\displaystyle\int_0^3 x^{6/5}\,dx$

7. $\displaystyle\int_1^\infty e^{-2s}\,ds$

8. $\displaystyle\int_{-\infty}^1 e^{4r}\,dr$

Let's summarize our knowledge of improper integrals and limits.

$$\int_a^\infty \frac{dx}{x^p} = \begin{cases} \underline{\hspace{3cm}} & \text{when } p \le 1 \\ \underline{\hspace{3cm}} & \text{when } p > 1 \end{cases}$$

$$\int_0^b \frac{dx}{x^p} = \begin{cases} \underline{\hspace{3cm}} & \text{when } p \ge 1 \\ \underline{\hspace{3cm}} & \text{when } p < 1 \end{cases}$$

$$\lim_{b\to\infty} b^p = \begin{cases} \underline{\hspace{3.5cm}} & \text{when } p < 0 \\ \underline{\hspace{3.5cm}} & \text{when } p > 0 \end{cases}$$

$$\lim_{x\to\infty} e^{kx} = \begin{cases} \underline{\hspace{3.5cm}} & \text{when } k < 0 \\ \underline{\hspace{3.5cm}} & \text{when } k > 0 \end{cases}$$

1. Do you think the integral $\int_1^\infty \frac{dt}{\sqrt{t^3+t}}$ converge or diverge? **Why?**

Analysis

2. We will try and answer this question by answering the following questions, and by developing this process, develop a technique for determining the convergence of all improper integrals we will see.

Q: Why is this integral improper?
A:

Q: What limit has to be taken to evaluate the integral?
A:

Q: What OTHER FUNCTION will the integrand **behave** like as we take this limit? (This is the hardest step. You have to decide how the integrand will change depending on the fact that, for example x is now a very very large number.)
A:

Q: Does the same type of improper integral with this OTHER FUNCTION as integrand converge or diverge? (Use the p-rules!)
A:

3. You use the above analysis to make your decision about whether you think the improper integral you're given converges or diverges and then you use the **Comparison Test for Improper Integrals** to **prove** your hunch.

Let's write down that proof here:

Comparison Test for Improper Integrals

This principle can be summarized as follows:

1. If $g(x) > f(x) > 0$ for all $x > a$ then if $\int_a^\infty g(x)dx$ CONVERGES, then $\int_a^\infty f(x)dx$ also CONVERGES.

2. If $f(x) > g(x) > 0$ for all $x > a$ then if $\int_a^\infty g(x)dx$ DIVERGES, then $\int_a^\infty f(x)dx$ also DIVERGES.

NOTE: You always compare the function and integral you're not sure about ($f(x)$) to the function you DO know about ($g(x)$).

Also you need to decide whether you are trying to prove convergence and divergence FIRST before you pick a function to compare to.
