Other Numerical Integration Techniques

1. (a) Sketch a (large!) graph of a function $f(x)$ that is decreasing and concave down over $[1, 5]$.

(b) Draw on your graph above four rectangles whose areas represent the Left Hand Riemann Sum approximation for $\int_1^5 f(x)\,dx$.
(c) Is LEFT(4) an underestimate or an overestimate for the definite integral? (LEFT(4) stands for the Left Riemann Sum with 4 rectangles).
(d) How about LEFT(10)?

(e) How about RIGHT(10)?

(f) How about MID(1)? MID(1) means the Riemann Sum with 1 interval, sampled at the MIDPOINT of each interval. (Sketch a new graph, if necessary).

(g) How about MID(2)?
MID(10)?

We don't necessarily have to use *rectangles* to approximate definite integrals. Why not use *trapeziods*?!

2. (a) Approximate the definite integral $\int_1^3 x^2 + 1\,dx$ using four trapezoids.

(b) Write expresions for LEFT(4) and RIGHT(4) for the above definite integral. Do not simplify.

(c) Which is larger, TRAP(4), or [LEFT(4) + RIGHT(4)]/2 ? Why?

For any function, TRAP(n)=

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3. Consider again the definite integral $\int_1^3 x^2 + 1 \, dx$.

(a) Does TRAP(4) give an overestimate or an underestimate?

(b) Is LEFT(4) a better estimate for the actual value of the definite integral, or RIGHT(4)? Why?

Does MID(4) give an overestimate or an underestimate?

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4. Now consider $\int_1^3 \frac{1}{x} \, dx$

(a) Does TRAP(4) give an overestimate or an underestimate?

(b) Does MID(4) give an overestimate or an underestimate?

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5. **Error in Numerical Integration**

Previously we had shown that whether LEFT or RIGHT is an under- or over-estimates depends on whether the function is INCREASING or DECREASING on the interval of integration. That is, the error LEFT or RIGHT makes depends on the DERIVATIVE of the function being integrated.

What feature of $f(x)$ do you think determines whether MID or TRAP gives an over- or under-estimate of the value of $\int_a^b f(x)dx$?