

**Trigonometric Examples Of Integration By Substitution**

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We now realize that the technique of Integration By Substitution comes from thinking about reversing the important Differentiation Rule, the Chain Rule. Just as the Chain Rule is nearly ubiquitous in helping you find derivatives, integration by substitution is very helpful in finding anti-derivatives.

**GROUPWORK**

One class of integrals which show up a lot in calculus are TRIGONOMETRIC INTEGRALS. In order to evaluate them we often have to recall a number of trigonometric identities.

$$\begin{aligned}\sin^2(x) + \cos^2(x) &= 1 \\ \tan^2(x) + 1 &= \sec^2(x) \\ \sin(2x) &= 2 \sin(x) \cos(x) \\ \cos(2x) &= \cos^2(x) - \sin^2(x) \\ \cos(2x) &= 1 - 2 \sin^2(x) \\ \cos(2x) &= 2 \cos^2(x) - 1\end{aligned}$$

To evaluate the following integrals you will need to use a trigonometric identity and then use integration by substitution.

1.  $\int \cos^2(x) dx =$

2.  $\int_0^{\pi/4} \sin^3(x) dx =$

**The Arctan Integral**

Consider  $\int \frac{1}{a^2 + x^2} dx$  and the substitution  $x = a \tan(u)$

1. Transform the integral from  $x$ -variables into  $u$ -variables. Can you write down an equation where  $u = f(x)$ ? What's the relationship between  $f(x)$  and  $\tan(u)$ ?

2. Find the anti-derivative.

3. Use your answer to compute the value of  $\int_0^1 \frac{dx}{3 + x^2}$