

**Examples of “Integral Exchange” using  $u$ -substitution**

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**Goal**

Our goal today is to have fun while we get more practice evaluating integrals using integration by substitution. We'll introduce the idea of thinking of integration by substitution as exchanging one integral (hard) for a different one (easy).

**Warm-up**

**Question:** How are the following integrals related?  $\int e^{\tan(x)} \sec^2(x) dx$  and  $\int 2xe^{x^2} dx$

**Answer:** We can think of two different  $u$ -substitutions which will make these integrals identical.

**GROUPWORK**

You will be split into groups of 3 or 4 students and assigned one of the integrals below.

First identify the  $u(x)$  function which will allow you to “exchange” the given integral for a simpler integral which you can evaluate easily.

Then evaluate your assigned integral by evaluating the simpler integral you “exchanged” it for.

Each group will then write their solution on the board so the rest of the class can follow each solution.

Each group will need someone with good handwriting to write-up the solution and another person who can explain the written solution and someone else who can answer questions about the solution.

Then we will go through the solutions together. Everyone should understand how to do all 6 integrals.

1.  $\int_2^4 2x^2(x^3 - 3)^{12} dx =$

2.  $\int_0^{\pi/2} \cos(2x)\sqrt{\sin(2x)} dx =$

$$3. \int_0^5 \sin(e^x) e^{x+\cos(e^x)} dx =$$

$$4. \int_0^1 \frac{x}{2x+1} dx =$$

$$5. \int_0^4 \frac{2x^2 - 4}{(x^3 - 6x + 20)^3} dx =$$

$$6. \int_1^8 \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}} dx =$$