Introduction to u-substitution

1. Find the following derivatives.

(a) $\frac{d}{dx}[(x^3+5x)^{14}] =$

(b) $\frac{d}{dx}[(\sin(x))^{24}] =$

(c) $\frac{d}{dx}[(u(x))^{32}] =$

2. Find the following indefinite integrals.

(a) $\int (3x^2+5)(x^3+5x)^{13}\, dx =$

(b) $\int \cos(x)[\sin(x)]^{23}\, dx =$

(c) $\int u'(x)[u(x)]^{31}\, dx =$

(d) $\int x^2(x^3+1)^{55}\, dx =$

This process involves identifying a composite function $g'(u)$ with an "inside function" $u(x)$ in your **integrand**.

In order to evaluate the integral using the Fundamental Theorem combined with The Chain Rule to find the anti-derivative the integrand must look like $g'[u(x)]u'(x)$.

Clearly: $\int g'(u(x))u'(x)dx = g[u(x)] + C$

This idea can be systematized into a technique of integration known as **INTEGRATION BY SUBSTITUTION**.

Identify the $g'(u)$ and $u(x)$ and $u'(x)$ in the following integral and thus evaluate it.

3. $\int \frac{\ln(x)}{x}\, dx =$

4. $\int (x+e^x)^9(1+e^x)\,dx =$

Let $u =$

Then $\frac{du}{dx} =$

Multiplying both sides by dx gives: $du =$

Now substitute into the original integral, so that everything is in terms of u instead of x.

5. $\int (x+e^x)^9(1+e^x)\,dx = \int$

This new integral should be easier than before. Solve it.

Now "convert back to x".

6. $\int \frac{t^2}{5+t^3}\,dt =$

Let $u =$

Then $\frac{du}{dt} =$

So $du =$

So $(\quad)\cdot du = t^2\,dt$

Substitute, then solve:

$\int \frac{t^2}{5+t^3}\,dt =$

Convert back:

How does this process change when you have a DEFINITE INTEGRAL?

7. $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$