More on Accumulation Functions

There are many examples of **accumulations** we have done in class so far (i.e. when we used the Subdivide-Approximate-Accumulate-Refine method of Riemann Sums to compute something).

One of the first was: DISTANCE TRAVELLED is the accumulation of SPEED with TIME.

Another accumulation can be written in words as

____________________ is the accumulation of ____________________

Using symbols, we can say that

____________________ $\approx$ ____________________

In general, we define an accumulation function as

$$A(\mathcal{X}) = \int_a^{\mathcal{X}} f(x)\,dx$$

In words, we would say that $A(\mathcal{X})$ is the accumulation of $f(x)$ with x, starting at a to some point $\mathcal{X}$.

Example 1

Let's look at some examples of accumulations that we can compute using geometry and other methods. Consider the graph of the following function $f(x)$:

Graphing Accumulation Functions

As $\mathcal{X}$ gets bigger, what does the graph of the accumulation of $f(x)$ from 0 to $\mathcal{X}$ look like? We can write this as $A(\mathcal{X}) = \int_0^{\mathcal{X}} f(x)\,dx$. Draw the graph of $A(\mathcal{X})$ on the axes below... (HINT: Try substituting in actual numbers for $\mathcal{X}$ to get values of $A(\mathcal{X})$ to be plotted.)

What if we start accumulating from 1 instead of zero? Sketch $B(\mathcal{X}) = \int_1^{\mathcal{X}} f(x)\,dx$ below. How is it different from $A(\mathcal{X})$?

Accumulation Functions as Anti-Derivatives

Let us define the accumulation function $A(\mathcal{X})$ for the *constant* function $f(x)$ as $A(\mathcal{X}) = \int_0^{\mathcal{X}} C\ dx$

1. As $\mathcal{X}$ gets bigger, what does the graph of the accumulation of the constant function $f(x) = C$ from 0 to $\mathcal{X}$ look like? Sketch it below (to the right).

2. Let us define another accumulation function $B(\mathcal{X})$ for the *linear* function $f(x) = x$ as $B(\mathcal{X} = \int_0^{\mathcal{X}} x\ dx =$. Sketch a graph of B on the axes below (to the right).

3. Do you see any relationships between the SLOPE of the graph of $A(\mathcal{X})$ at some point $\mathcal{X}$ and the corresponding value $f(\mathcal{X})$?

4. How would the graphs change if you started accumulating from 1 instead of 0?