Accumulation Functions and Definite Integrals

Signed Area

NOTE: the value of $\int_a^b f(x)\,dx$ is is not always the exact same number as the size of the area A between the function $f(x)$ and the x-axis.

In each of the following cases write down expressions involving the definite integral of the function $f(x)$ between a and b

A. When $f(x) \geq 0$ over the interval of integration $[a, b]$,

B. When $f(x) \leq 0$ over the interval of integration $[a, b]$,

C. When $f(x) \geq 0$ from $a \leq x \leq c$ and $f(x) \leq 0$ from $c \leq x \leq b$,

One can compute a definite integral by knowing the area it represents.

EXAMPLE Find $\int_{-2}^{4} 5\ dx$.

Accumulation Functions

Recall the function $L(b) = \int_1^b \frac{1}{x}\, dx$ from the lab. In words, $L(b)$ represents the (signed) area between the curve $f(x) = 1/x$ and the lines $x = 1$ and $x = b$, where $b > 0$.

Exercises

Use the graph above to assist you in answering the following questions in small groups.

1. What is the value of $L(1)$? (Write it out as a definite integral and consult the graph.)

2. On the graph, indicate $L(2)$ (Write it out as a definite integral and consult the graph).

3. Which is larger $L(3)$ or $L(5)$? (Write them out as definite integrals and consult the graph).

4. In fact, can you determine whether $L(b)$ is monotonically INCREASING or monotonically DECREASING on $[1, \infty)$? **Explain your answer.**

5. As b gets larger, does the rate of change of $L(b)$ get smaller or larger? In other words, for larger values of b is more and more area being accumulated or less? Another way to think about this woudl be to think about what $L'(b)$ looks like as b increases....

The function $L(b)$ is called an **accumulation function** for $1/x$.
Let's use the answers to the questions above to graph $L(b)$ versus b for $b > 1$ on the axes above.

DEFINITION

The formal definition of an **acumulation function** $F(\mathcal{X})$ of a function $f(x)$ on the interval $[a, \mathcal{X}]$ is given by:

$$F(\mathcal{X}) = \int_a^{\mathcal{X}} f(x)dx$$