Area, Riemann Sums and Summation Notation

1. Sigma Notation.

$$\sum_{i=1}^{5} i =$$

$$\sum_{i=3}^{11} \frac{1}{i} =$$

$$\sum_{j=-3}^{5} (j+10) =$$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} =$$

2. Using Riemann Sums To Compute Areas

Suppose we want to find the area A of the region described below: $0 \le x \le 2$, above the x-axis, and below $f(x) = x^2 + 1$.

(a) Sketch and shade the region described above.

(b) In the shaded region draw two rectangles of equal width, neither of which goes above the curve $f(x) = x^2 + 1$.

(c) The width of each rectangle is _____.
The heights of the two rectangles are _____ and _____.
The sum of the areas of the two rectangles is _____.

(d) Find a better estimate for the area of the shaded region A by repeating the above procedure, but this time with FOUR rectangles.

(e) Repeat with 100 rectangles! Just write the appropriate expression, but do not evaluate it. This expression is called a **Riemann Sum**. The general form of a Riemann Sum for a given function $f(x)$ evaluated at points separated by Δx is $\sum_{k=1}^{n} f(x_k)\Delta x$.

(f) Repeat with n rectangles, instead of 100.

Can you see how this expression has the form of a Riemann Sum?

(g) The larger n is, the more __________ this estimate is. So, the EXACT value of the area can be written down as:

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3. Using Other Sample Points.

On this worksheet, we have been doing **left-hand** Riemann Sums. This means:

> In each "sub-interval" we used the ______ endpoint to calculate the height of each rectangle.

There was no special reason to use LEFT-hand sums. We could just as well use RIGHT-hand Riemann Sums, and everything would still work the same (except in this example we'd happen to get over-estimates). In fact, as long as you evaluate the function at SOME POINT in each sub-interval and multiply by the size of that sub-interval and add up your products we still get a Riemann Sum.

IF, in the LIMIT, the Riemann sums give the same exact number regardless of sample point THEN this number is called the value of **the definite integral**.

(We'll show that this is true in a number of cases in lab.)

GroupWork
Compute the right-hand Riemann Sum approximation for the area A, using two rectangles, and then again using four rectangles.

Let's also use the midpoint Riemann Sum to obtain approximations for the area A, using four rectangles.