Computing traveled distance from velocity

1. (a) Suppose you are driving your car at a constant velocity of 30 miles per hour. What distance do you travel in one hour? _____

(b) What distance do you travel in one minute? _____

(c) Now suppose at 8:30:00 you start to accelerate. Each second, you go 0.2 miles/hour faster. So, your velocity at 8:30:05 is _____, and at 8:31:00 is _____.

(d) Let D be the distance you traveled from 8:30:00 to 8:31:00. Is D greater or smaller than $30 \times (1/60)$?

(e) Is D greater or smaller than $42 \times (1/60)$?

Why? Explain your answer!

2. (a) To get a better estimate for D, do the following steps.

Step 1. Complete the following table.

TABLE 1			
Time from 8:30:00 (in seconds)	0	30	60
Velocity (in miles/hour)			
Distance traveled (in miles)			

Step 2. Use your results in the above table to approximate D.

(b) Is this an over-estimate or an under-estimate for D? Why?

3. In Table 1 above, we used the INITIAL velocity for each time interval, i.e., the velocity at the beginning of each time interval.

(a) Complete the table below using the TERMINAL velocity for each time interval.

TABLE 2			
Time from 8:30:00 (in seconds)	0	30	60
Terminal Velocity (in miles/hour)			
Distance traveled (in miles)			

(b) Use your results in the above table to approximate D.

(c) Is this an over-estimate or an under-estimate for D? Why? This is called the RIGHT-HAND SUM for the distance, while our calculation from Table 1 is called the _____ for the distance.

4. (a) To get a more accurate estimate, repeat the same procedure, but for time intervals that are half as long, i.e., ____ seconds instead of ____ seconds, each. (You may use initial or terminal velocities, whichever you prefer.)

TABLE 3						
Time from 8:30:00 (in seconds)						
Velocity (in miles/hour)						
Distance traveled (in miles)						

(b) Use your results in Table 3 to approximate D, the distance travelled. Are you using a left-hand sum or a right-hand sum? (You might want to review your work in Table 1 and Table 2.)

5. To get increasingly more accurate estimates for D, the distance traveled from 8:30:00 to 8:31:00, we could divide up the 1 minute into more time intervals.

With 3 time intervals, each time interval is $\Delta t = \frac{60}{\quad}$ seconds.
Let v_1, v_2, v_3 represent the velocities we used for these 3 time intervals in Table 1.
Then we can write:

$D \approx \Delta t \cdot v_1 +$

SIMILARLY, with n time intervals, each time interval would be $\Delta t = \frac{60}{\quad}$ seconds.
Let $v_1, v_2, v_3, \cdots, v_n$ represent the velocities for these n time intervals.

Then we can write:

$D \approx \Delta t \cdot v_1 +$

6. The larger n is, the _____ this estimate is. So as we use larger and larger values for n, and repeat this entire calculation for each n, we get successively more accurate approximations for D.

Thus, the EXACT value of D is written as:

$$D = \lim_{n \to \infty} [\Delta t \cdot v_1 + \cdots + \Delta t \cdot v_n]$$