Class 25: Friday October 31

Periodic Motion of a Spring

We're going to try and derive an initial value problem for the motion of a spring.

Let's assume that we have a weight of mass m and denote the default or "rest" position of the spring as $x = 0$. If the spring moves up vertically, then $x > 0$, when it moves below the rest position $x < 0$.

Let's assume that the spring force is proportional to the amount the spring is displaced.

1. Write down an equation which relates F the force of the spring and x the displacement.

Newton's Law of Motion states that the force acting on a body is a product of its mass and acceleration. We also know that acceleration is the rate of change fo velocity with time and velocity is the rate of change of distance with time.

$$F = ma = m\frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Let's assume that one initially holds the spring motionless at some point $x = a$ above its rest position. Putting this information together, we obtain the following initial value problem for x and v

$$\begin{aligned} x' &= v & x(0) &= a \\ v' &= -b^2 x & v(0) &= 0 \end{aligned}$$

We can also think of this is a second order differential equation for $x(t)$.

$$x'' = -b^2 x, \qquad x(0) = a, \qquad x'(0) = 0$$

[Notice it has two initial conditions for $x(t)$.] From the lab we know that $\cos(bt)$ and $\sin(bt)$ have the property that $(\cos(bt))'' = -b^2\cos(bt)$ and $(\sin(bt))'' = -b^2\sin(bt)$.
2. Can we guess the solution of this initial value problem for x(t)?
(a) Try and find A and ω such that $f(t) = A\cos(\omega t)$ solves the IVP for the spring.

(b) Try and find B and ω such that $g(t) = B\sin(\omega t)$ solves the IVP for the spring.