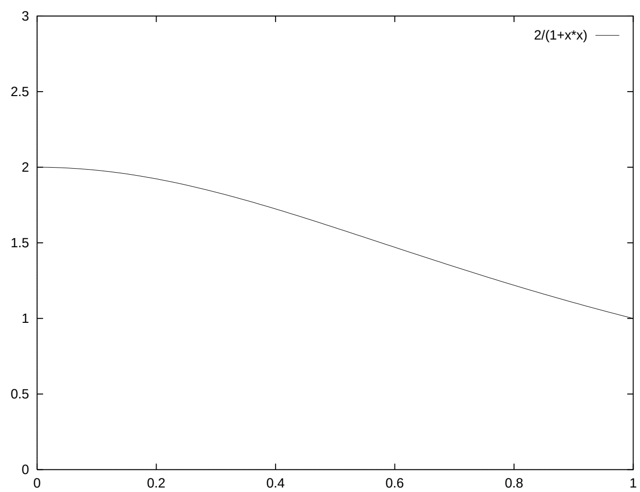
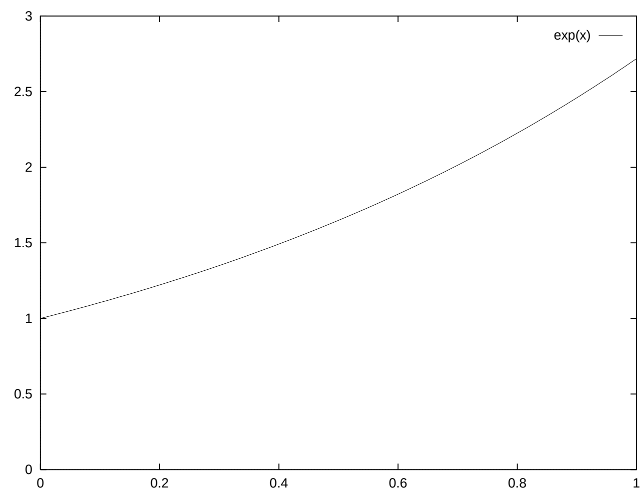
Class 23: Monday October 27

Error Analysis of Numerical Integration

Today we will be trying to obtain expressions for the relationship between $I = \int_a^b f(x)dx$ and A the numerical approximation to the value of I. Often what we are really interested is $E = |A - I|$ the absolute error in the numerical approximation.

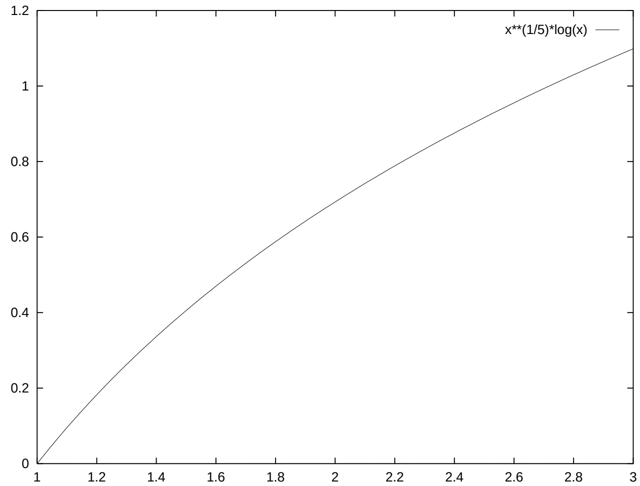


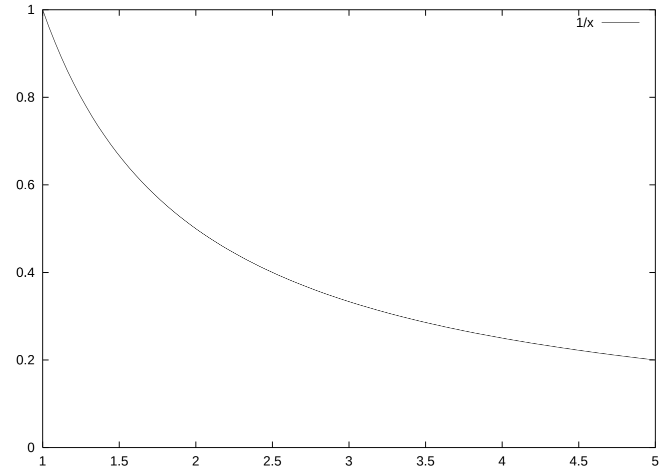


We will use *Left and Right Riemann Sums* to approximate the area "under the curves" in the figures. We will use *Error Stacks* to bound the error of our approximation.

Riemann Sums and Error Stacks

Thus we can show that for a Riemann sum $E = f'(c)(b-a)\Delta x$ and E is proportional to ____________ and ____________.





We know that the error due to trapezoid and midpoint depend on the ____________________ of the curve.
We might have noticed that the error decreases as ____________________ *increases.*
We also know that the error due to midpoint is always less than trapezoid.
It turns out that the expressions for these errors look like:

$$|I - T| = f''(c)(b-a)\frac{h^2}{12} = f''(c)\frac{(b-a)^3}{12N^2}$$

$$|I - M| = f''(c)(b-a)\frac{h^2}{24N^2} = f''(c)\frac{(b-a)^3}{24N^2}$$

$$|I - S| = f^{(4)}(c)(b-a)\frac{h^4}{2880} = f^{(4)}(c)\frac{(b-a)^5}{2880N^4} \text{ where } h = \frac{b-a}{N} = \Delta x.$$

Error Control

We can use our knowledge of how the error in our approximation depends on N and h to determine how large N would have to be to get a certain error.

- On the interval between $x = 4$ and $x = 13$, the function $f(x)$ is decreasing. $f(4) = 212$ and $f(13) = -8$. Find the size of Δx needed to ensure that any Riemann sum using that Δx is within 0.1 of the actual value. Then find the number of subintervals n needed.

- On the same interval we somehow know that f'''' is always less than 20. How large would N have to be to obtain a maximum error of 0.0001 when using Simpson's Method?