Math 118 **ADVANCED PLACEMENT CALCULUS** Fall 2003

Class 17: Monday October 13

Introduction to Accumulation, Accumulation Functions and Integration

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Velocity and Distance

Suppose you are driving on back roads through the Rocky Mountains and neither the odometer nor the gas guage are working in your vehicle. Fortunately, the speedometer is working and you are driving with a friend who has a watch and who has some facility with paper and a pencil. Next we examine how a sequence of speedometer readings, say, one speedometer reading each minute, along with the relationship between distance and velocity, can be used to approximate the distance you have travelled.

1. If you drive with constant velocity v feet/sec for Δt seconds, how far have you gone?

2. Suppose $v(t)$ is not constant, but that $v(t)$ changes very little on the time interval $T \leq t \leq T + \Delta t$. Approximately how far have you driven during this time interval?

3. If you drive a distance s_1 and then drive a distance s_2, how far have you driven altogether?

4. Use the ideas we have considered in this class so far, along with your answers to the questions above, to estimate the distance travelled by a car whose velocity satisfies the following table. Obtain both an *underestimate* and an *overestimate* of this distance.

Time (sec)	0	1	2	3	4	5
Velocity (feet/sec)	20	30	38	44	48	50

5. How could your estimates be improved?

Riemann Sums

Suppose a function f is defined on an interval $a \leq t \leq b$. Suppose the interval is divided up into n subintervals of lengths $\Delta t_1, \ldots, \Delta t_n$. Suppose t_k is a point chosen from the subinterval Δt_k for each $k = 1, 2, \ldots, n$. A **Riemann sum** for f on the interval $[a, b]$ is a sum of the form

$$\sum_{k=1}^{n} f(t_k)\Delta t_k = f(t_1)\Delta t_1 + f(t_2)\Delta t_2 + \cdots + f(t_n)\Delta t_n.$$

If the *left* endpoint of each subinterval is chosen, the Riemann sum is a **left-hand sum**.

If the *right* endpoint of each subinterval is chosen, the Riemann sum is a **right-hand sum**.

Riemann Sums give us approximations to what property of the function $f(x)$?

Example 1: Velocity to Distance

Suppose $v(t)$ is a function that describes the velocity of a moving particle, in meters per second, at a time t seconds after some starting time $t = 0$. What is the interpretation of the accumulation function $D(x)$ which is the **accumulation of velocity with time**?

Assume that the velocity of a moving particle is given by the following piecewise linear function:

$$v(t) = \begin{cases} 10, & \text{for} \quad 0 \leq t < 5 \\ 2t, & \text{for} \quad 5 \leq t < 40 \\ 80, & \text{for} \quad 40 \leq t < 50. \end{cases}$$

Determine the appropriate formulas which define the accumulation function $D(x)$ over the three intervals:

$$D(x) = \begin{cases} & \text{for} \quad 0 \leq x < 5 \\ & \text{for} \quad 5 \leq x < 40 \\ & \text{for} \quad 40 \leq x < 50. \end{cases}$$

Example 2: Electrical Power and Energy

The work done by electrical *power* over some period of time, in illuminating light-bulbs for example, is referred to as electrical *energy*; energy = power $\times$ elapsed time. If power is measured in megawatts and time in hours, electrical energy is measured in megawatt-hours (MWh). See the graph below, from CiC p. 354.

Make a table with headings T hours and $E(T)$ MWh. Here $E(T)$ is the total accumulated energy between 0 hours and T hours. Using the graph of the power function above, evaluate $E(T)$ for T = 0, 6, 9.5, 15, 21, and 24 hours. Then plot $E(T)$.

What is the relationship between $E(T)$ and $p(t)$? Is this similar or different from the relationship between $D(x)$ and $v(t)$

We can say that ____________ and ____________ are examples of **ACCUMULATION FUNCTIONS**.

DEFINITIONS.

The definite integral. This evaluates to a *number*. For example,

$$\int_2^5 f(x)dx = \lim_{n\to\infty}\sum_{k=1}^n f(x_k)\Delta x, \qquad 2 \le x_1 < \ldots < x_n \le 5, \quad \Delta x = (5-2)/n.$$

Now realizing the specific upper limit of integration, 5, in the definite integral may be replaced by a variable gives us an **accumulation function**:

$$F(x) = \int_2^x f(t)dt = \lim_{n\to\infty}\sum_{k=1}^n f(t_k)\Delta t, \qquad 2 \le t_1 < \ldots < t_n \le x, \quad \Delta t = (x-2)/n.$$

Note that the variable x used for the interval endpoint is *not* the same as the "dummy variable" t appearing in the integral. Replacing x with a particular number specifies an interval of integration. Provided f is (Riemann) integrable on this interval, the corresponding definite integral can be evaluated to produce a unique number. Each input to the accumulation function determines a unique output value. In this context, our geometric interpretation of the definite integral helps us to see that the formula

$$F(x) = \int_a^x f(t)\,dt,$$

defines a function with independent variable x, by giving us an intuitive understanding of the output of the function: if any specific value of x is plugged in, say $x = 4$, then $F(4)$ is the signed area trapped between the curve $f(t)$ and the x-axis from $x = a$ to $x = 4$.

Write down an expression which relates the distance travelled $D(x)$ and the velocity $v(t)$

Write down an expression which relates the electrical energy used $E(T)$ and the power function $p(t)$

What are the differences between an accumulation function and a definite integral?