Class 11: Wednesday September 24

Connections between Derivatives and Graphs of Functions

GroupWork

Today's class will be an interactive session to try to stimulate your memory on the connections between the first and second derivatives of a function, and the behavior of the function. In groups of 2 or 3, answer the following questions below, and then we will go over the answers. Each group will answer at least one question for the entire class, so make sure you have one person who is recording the concensus knowledge of the group (the Recorder), and another person willing to report the results to the entire class (The Reporter).

Let f be a differentiable function, i.e., f is locally linear everywhere. Explain your answers to the following, using graphs to illustrate your reasoning.

1. If $f'(1) = 0$ and $f'(2) = 0$, can we conclude that $f(x) = 0$ for some $x \in (1, 2)$?

2. If $f'(2) = 0$, can we conclude that f has a local max or min at $x = 2$?

3. If $f'(x)$ changes sign from positive to negative at $x = 2$, can we conclude that f has a local maximum at $x = 2$? Can we conclude that f has an absolute maximum at $x = 2$?

4. What can we say about f at $x = 1$ if $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$?

5. What can we say about $f'(x)$ for $x \in [-5, 0]$ if $f(x)$ is decreasing on this interval?

6. What can we say about $f(x)$ for $x \in [3, 7]$ if $f'(x) > 0$ on this interval?

7. If $f(-2) = 0$ and $f(-1) = 0$, can we conclude that $f'(x) = 0$ for some $x \in (-2, -1)$?

8. If $f''(2) = 0$, can we conclude that f has an inflection point at $x = 2$?

9. What can we say about f for $x \in [-6, -5]$ if $f''(x) < 0$ on this interval?

10. What can we say about f' for $x \in [3, 5]$ if $f''(x) < 0$ on this interval?

11. Can we have a function f that is increasing and concave down? What about decreasing and concave up?

12. If $f'(4) = 0$ and $f''(4) = -3$, can we conclude that f has a local max at $x = 4$? Can we conclude that f has an absolute max at $x = 4$?

13. What can we say about $f''(x)$ for $x \in [4, 8]$ if $f(x)$ is concave up on this interval?

14. If $f'(8) = 0$ and $f'(9) = 0$, can we conclude that $f''(x) = 0$ for some $x \in (8, 9)$?

15. If $f'(8) = 0$ and $f'(9) = 0$, can we conclude that f has an inflection point at some $x \in (8, 9)$?