

Math 118 Homework #6
Fall Term 2003

Directions: Complete the work for these questions on scratch paper, and then fill in the blank spaces on this sheet. Use complete sentences and organized calculations in your responses.

Approach to Steady State

1. A physician decides to give a patient an infusion of glucose at a rate of c grams per hour. The body of the patient converts the glucose and removes it from the body at a rate proportional to the amount present in the bloodstream, say at r grams per hour per gram of glucose present.

- a. We model the amount $G = G(t)$ of glucose present at time t by a differential equation of the form

$$\frac{dG}{dt} = c - rG.$$

Why? How does this fit with the statement above? Also verify that the units are correct.

- b. Given an initial amount G_0 of glucose in the bloodstream at time 0, find an explicit formula for the amount of glucose present at any time t . Hint: compare this problem with Newton's Law of Cooling, and note that we have an IVP that can be written as :

$$\begin{aligned}\frac{dG}{dt} &= -r\left(G - \frac{c}{r}\right) \\ G(0) &= G_0\end{aligned}$$

- c. The level of glucose in the patient will reach an equilibrium level (the steady state) for given values of c and r . Explain how to compute the steady state from the differential equation in part a. Also explain how you could determine the same value from your solution in part b. What is the steady state level of glucose? Note: these two methods of finding the steady state should produce the same value. If they do not, there is an error somewhere that you need to fix.

2. A physician orders an infusion of 10 grams of glucose per hour for a patient. Lab technicians determine that the patient has 2 grams of glucose in his bloodstream and that his body will remove glucose from the bloodstream at a rate of 3 grams per hour per gram of glucose.

- a. How much glucose will be in the bloodstream two hours after the infusion is started?

- b. How long will it take for the glucose level to reach 3 grams?

3. Suppose a patient's bloodstream has 2 grams of glucose, and her physician wants to raise this amount to 3.5 grams in three hours. It is determined that her system removes glucose from the bloodstream at a rate of 4 grams per hour per gram of glucose. How fast should the physician order the glucose to be infused into the patient's bloodstream?