**NAME:** ______________________________

Solutions to the Linear Spring Problem $x'' = -b^2 x$.

This worksheet gives the opportunity to work with differential equations of the form $x'' = -b^2 x$ and their solutions. Recall that this equation arose from our discussion of the linear spring. We will see what the general solution looks like.

The structure of this exercise is a number of different examples. In each section, some of the expressions are filled in while others are blank. Your job is to fill in the blanks and make sure you check each condition.

1. (This example has everything filled in. Check all the conditions.)

| | | |
|---|---|---|
| $x(t) = 7\cos(3t)$ | $x(0) = 7$ | Check: $x(0) = 7\cos(0) = 7$ ✓ |
| $x'(t) = -21\sin(3t)$ | $x'(0) = 0$ | Check: $x'(0) = -21\sin(0) = 0$ ✓ |
| $x''(t) = -63\cos(3t)$ | $x'' = -3^2 x$ | Check: $x'' = -9 \cdot 7\cos(3t)$ ✓ |

2. (Refer to example 1.)

| | | |
|---|---|---|
| $x(t) =$ ____ $\cos(3t)$ | $x(0) = 17$ | Check: ______________________ |
| $x'(t) =$ ______________________ | $x'(0) = 0$ | Check: ______________________ |
| $x''(t) =$ ______________________ | $x'' = -3^2 x$ | Check: ______________________ |

3. (Use the previous examples as a guide.)

| | | |
|---|---|---|
| $x(t) =$ ______________________ | $x(0) = a$ | Check: ______________________ |
| $x'(t) =$ ______________________ | $x'(0) = 0$ | Check: ______________________ |
| $x''(t) =$ ______________________ | $x'' = -3^2 x$ | Check: ______________________ |

4. (Compare with the previous examples. What has changed?)

| | | |
|---|---|---|
| $x(t) = 8\sin(3t)$ | $x(0) = 0$ | Check: ______________________ |
| $x'(t) =$ ______________________ | $x'(0) = 24$ | Check: ______________________ |
| $x''(t) =$ ______________________ | $x'' = -3^2 x$ | Check: ______________________ |

5. (Use example 4.)

| | | |
|---|---|---|
| $x(t) =$ ____ $\sin(3t)$ | $x(0) = 0$ | Check: ______________________ |
| $x'(t) =$ ______________________ | $x'(0) = p$ | Check: ______________________ |
| $x''(t) =$ ______________________ | $x'' = -3^2 x$ | Check: ______________________ |

6. (This is like a combination of examples 1 and 4.)

| | | |
|---|---|---|
| $x(t) = 7\cos(3t) + 8\sin(3t)$ | $x(0) = 7$ | Check:__________ |
| $x'(t) =$ __________ | $x'(0) = 24$ | Check:__________ |
| $x''(t) =$ __________ | $x'' = -3^2x$ | Check:__________ |

7. (Same idea as 6.)

| | | |
|---|---|---|
| $x(t) =$ ____ $\cos(3t) +$ ____ $\sin(3t)$ | $x(0) = 17$ | Check:__________ |
| $x'(t) =$ __________ | $x'(0) = 24$ | Check:__________ |
| $x''(t) =$ __________ | $x'' = -3^2x$ | Check:__________ |

8. (Same idea as 6 and 7.)

| | | |
|---|---|---|
| $x(t) =$ ____ $\cos(3t) +$ ____ $\sin(3t)$ | $x(0) = a$ | Check:__________ |
| $x'(t) =$ __________ | $x'(0) = 24$ | Check:__________ |
| $x''(t) =$ __________ | $x'' = -3^2x$ | Check:__________ |

9. (Same idea as 6-8, we just added another parameter.)

| | | |
|---|---|---|
| $x(t) =$ ____ $\cos(3t) +$ ____ $\sin(3t)$ | $x(0) = a$ | Check:__________ |
| $x'(t) =$ __________ | $x'(0) = p$ | Check:__________ |
| $x''(t) =$ __________ | $x'' = -3^2x$ | Check:__________ |

10. (A DIFFERENT two-parameter problem. What was changed?)

| | | |
|---|---|---|
| $x(t) =$ ____ cos( ____ $t$) + ____ sin( ____ $t$) | $x(0) = a$ | Check:__________ |
| $x'(t) =$ __________ | $x'(0) = p$ | Check:__________ |
| $x''(t) =$ __________ | $x'' = -5^2x$ | Check:__________ |

11. (A three-parameter problem. This is the most general form!)

| | | |
|---|---|---|
| $x(t) =$ ____ cos( ____ $t$) + ____ sin( ____ $t$) | $x(0) = a$ | Check:__________ |
| $x'(t) =$ __________ | $x'(0) = p$ | Check:__________ |
| $x''(t) =$ __________ | $x'' = -b^2x$ | Check:__________ |

12. (One final try.)

| | | |
|---|---|---|
| $x(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ | $x(0) = -1$ | Check:____________________ |
| $x'(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ | $x'(0) = -\sqrt{3}$ | Check:____________________ |
| $x''(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ | $x'' = -1^2 x$ | Check:____________________ |

13. Use what we learned before about writing the sum of a sine and cosine function as a shifted cosine function to rewrite the $x(t)$ from 12 in an alternate form (as a shifted cosine function). You may need to look up the method again.

14. We should see now that the general form of the solution to the initial value problem

$$\begin{aligned} x'' &= -b^2 x \\ x(0) &= a \\ x'(0) &= p \end{aligned}$$

is of the form

$$x(t) = A\cos bt + B\sin bt.$$

It is easy to solve for the constants $A$ and $B$ by using the initial conditions.

$$x(0) = \_\_\_\_\_\_\_\_\_\_\_\_ = a$$

$$x'(t) = -bA\sin bt + bB\cos bt$$

$$x'(0) = \_\_\_\_\_\_\_\_\_\_\_\_ = p$$

So

$A =$ ____, $B =$ ____, and $x(t) =$ ____________________________________.

15. Use this method to find the solution to

$$\begin{aligned} x'' &= -25x \\ x(0) &= 4 \\ x'(0) &= -10 \end{aligned}$$