

Hares and Foxes (based on a problem in *CiC*)

The interaction of Hares and Foxes can be modeled using a *predator-prey* model.

Simplifying assumptions:

- Without foxes around, the hares would grow exponentially.
- Without hares around, the foxes would die out.
- Interactions between foxes and hares benefit the foxes.

Notation:

H = number of hares in hundreds.

F = number of foxes in hundreds.

Rates of change: $H'(t)$, $F'(t)$.

Units: numbers of hares/foxes per month.

The system of equations looks like:

$$H'(t) = aH(t) - bH(t)F(t)$$

$$F'(t) = bH(t)F(t) - cF(t)$$

where a, b, c are all positive.

1. For each assumption, indicate the corresponding part of the equations.

2. Given:

$$H'(t) = 0.3H(t) - 0.2H(t)F(t)$$

$$F'(t) = 0.2H(t)F(t) - 0.1F(t).$$

Use Eulers method to calculate the population sizes when $t = 200$ for $H(0) = 5$, $F(0) = 5$.

Use a step size, $\Delta t = .1$. Plot the results (include the graph in your homework). Interpret the results you have visualized, i.e. what is happening to the populations?

3. Repeat problem (2) for $H(0) = 0.5$ and $F(0) = 1.5$. What happens? Why does it happen? (If you are not sure, calculate the rates of change at the initial conditions by hand.)