Population Models and Qualitative Analysis, Part 2

We have already seen two models of population growth. As you can imagine, there are many more ways to model population growth. In this homework, you will consider a model that is often used to model tumor cell populations. This model is known as the **Gompertz growth** model. This model is not an extension of the logistic or exponential models. According to the Gompertz growth model, a population of say tumor cells, T, changes with time according to:

$$\frac{dT}{dt} = -\kappa T \ln T.$$

Whereas the exponential growth model is often applicable to populations at small sizes, this model captures behavior of larger populations better. *What is a mathematical reason why the Gompertz model may not work well for small population values?*

Sketch the graph of the slope function versus T. Use this graph to sketch possible solutions $T(t)$ versus time, t.

Are there any equilibrium values in the Gompertz growth model? If so, find all equilibrium values T^* and explain how the equilibrium values are related to the graphs you plotted above. How do these equilibrium values compare to those we saw for the exponential and logistic population models?

What is the relative growth rate? What does it tell us about how individuals in the population reproduce?

Verify that

$$T(t) = e^{\ln(T_0)e^{-\kappa t}}$$

satisfies the initial value problem for Gompertz growth, given by:

$$\begin{aligned} \frac{dT}{dt} &= -\kappa T \ln T \\ T(0) &= T_0 \end{aligned}$$

How well do you think this model might work in capturing certain population growth over time? What are some pros and cons of the model?

While we have looked at increasingly complicated population models, there are many more complex models out there. We did not discuss an extremely simple population model. Can you think of what it might be and why it isn't really used?