The Product and Quotient Rules

Warm-Up

Suppose both f and g are differentiable at a. Use Taylor's Theorem to rewrite each of the following in the form, "linearization about a, plus error."
(Use $E_1(h)$ to denote the error for f and $E_2(h)$ to denote the error for g.)

$$f(a+h) =$$

$$g(a+h) =$$

The Product Rule

Suppose f and g are both differentiable at a. Let $p(x) = f(x)g(x)$. Then

$$p'(a) = f'(a)g(a) + f(a)g'(a)$$

Proof. (Use the definition of the derivative and Taylor's Theorem to prove that the product rule is correct.)

Practice With the Product Rule

Find the derivatives of the following functions. In some cases it may be useful to factor before differentiating.

2. $f(x) = (3x^4 + x^2 + 1)\, 2^x$

3. $g(t) = t^{40} \cdot \cos t$

4. $h(s) = \sin^2(s)$

5. $p(r) = e^{2r}\,(r^4 + \tan r)$

Quotient Rule

Suppose f and g are both differentiable at a, and that $g(a) \neq 0$. Let $Q(x) = f(x)/g(x)$. Then

$$Q'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{[g(a)]^2}.$$

6. Assuming the hypotheses of this theorem are true, confirm that

$$f(a) = Q(a)g(a).$$

Then, assuming Q is differentiable at a, differentiate both sides of this equation. Use the Product Rule to differentiate the right-hand side. Then solve for $Q'(a)$.

This *proves* that the quotient rule is correct provided Q is differentiable at a. (Next week we will be able to confirm that Q *is* differentiable at a by using the Chain Rule.)

Find the derivative of the following function:

7. $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

Note: To simplify your answer, recall that $\sec(x) = 1/\cos(x)$.