## Introduction to the S-I-R Model of Disease

Suppose we want to model the spread of an infectious disease (like measles).

Simplifying assumptions:
– Nobody dies from it!
– Recovery always takes 14 days.
– You're contagious during those 14 days.
– You cannot get it twice.

*Notation:*
$I$ = number of infected people.
$R$ = number of recovered people (i.e., already had it).
$S$ = number of susceptible people (i.e., haven't had it yet).

Rates of change: $I'(t)$, $R'(t)$, $S'(t)$.
Units: ________________ per day.

Q: If $I$ people are currently infected, how many of them do you expect will recover today? ______.

So,

$$R'(t) =$$

True of false?
$I'(t)$ = number of people who get infected per day.

$S'(t)$ = -(number of people who get infected per day).

To write an equation for $S'(t)$, first note that on any given day, the number of people who get infected depends on the number of susceptible people who come into contact with infected people:
–If everything else was the same except there were twice as many *susceptible* people, how would this affect the number of people who *become infected*?

So,

$$S'(t) \text{ is proportional to}$$

–If everything else was the same except there were twice as many *infected* people, how would this affect the number of people who *become infected*?

So,

$$S'(t) \text{ is proportional to}$$

These combine to give

$$S'(t) =$$

*Why multiplied?* Let's think about it from a different perspective. If $k$ is the proportional number of contacts that an infected person has, and $S(t)$ is the number of susceptible people, then

The rate of people beoming infected (for every infected person) is

If we multiply this by the number of infected people, we get:

What about $I'(t)$ ? It should equal
(number of people who get infected per day) − (number of people who ____________________).

So,

$$I'(t) =$$

Given

$$\begin{aligned} S' &= -0.00001SI \\ I' &= 0.00001SI - (1/14)I \\ R' &= (1/14)I \\ S_0 = 35000 \qquad I_0 &= 100 \qquad R_0 = 4900 \end{aligned}$$

$\boxed{\text{Groupwork}}$

**1.** In small groups, try to develop IVPs for the following situations which modify the given S-I-R model above.

a. **Vaccination.** A modification of the original SIR model above after a partially successful vaccine is given to the population which cuts the infectiousness down to one quarter of its present infectiousness.

b. **Improve treatment.** A modification of the original SIR model after a treatment is discovered which reduces the time one is sick to 3 days.

c. **Immunity Loss.** A modification of the original SIR model so that 1 out of every 200 persons who recover become susceptible again.

d. **Death.** A modification of the initial SIR model so that 1 out of every 30 persons who are infected dies, the rest recover.