Lab Time: **Your Name:** BUCKMIRE

a. *(5 points.)* The function $f(x)$ below is not continuous at $x = 2$:

$$f(x) = \begin{cases} \dfrac{1+\sin(\pi x)}{\sqrt{x+7}}, & x \neq 2 \\ 3, & x = 2 \end{cases}$$

Use the information above to evaluate the following limit:

$$\lim_{x\to 2} f(x) = \frac{1}{3} = \frac{1+\sin(2\pi)}{\sqrt{2+7}} = \frac{1+0}{\sqrt{9}} = \frac{1}{3}$$

Explain your answer.

You know $f(2)=3$ and $f(x)$ is discontinuous so $\lim_{x\to 2} f(x)$ can NOT be 3. Since $\dfrac{1+\sin(\pi x)}{\sqrt{x+7}}$ is a ~~combination of~~ continuous function (except for $x=-7$) then

$\lim_{x\to 2^-} f(x) = \frac{1}{3}$ and $\lim_{x\to 2^+} \dfrac{1+\sin \pi x}{\sqrt{x+7}} = \frac{1}{3}$ so $\lim_{x\to 2} f(x) = \frac{1}{3}$

b. *(5 points.)* Explain why the following function is (or **is not**) continuous at $h = 0$. (You do not need to evaluate a limit or do much calculation to answer this question.)

$$Q(h) = \frac{(3+h)^{100} - 3^{100}}{h}$$

Q is not continuous at $h=0$ because $Q(0)$ is undefined. For continuity $\lim_{h\to 0} Q(h)$ must equal $Q(0)$. Since $Q(0)$ doesn't exist, regardless of what $\lim_{h\to 0} Q(h)$ is, $Q(h)$ is discontinuous at $h=0$.

2. **BONUS** *(5 points.)* Obtain a relatively simple expression for $\lim_{h\to 0} Q(h)$.

$$\lim_{h\to 0} Q(h) = \lim_{h\to 0} \frac{(3+h)^{100} - 3^{100}}{h}$$

Looks like a difference quotient!

$$= \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

where $f(x) = x^{100}$

$$= f'(3)$$

$f'(x) = 100x^{99}$

$$= 100 \cdot 3^{99}$$