SHOW ALL YOUR WORK

(10 points) Reconsider the rate equation $\frac{dx}{dt} = (x-2)(x+1)$. Verify that the solution $x(t)$, implicitly given by

$$\frac{x-2}{x+1} = \frac{1}{4}e^{3t}$$

satisfies the I.V.P. (initial value problem) consisting of the rate equation above and the initial condition $x(0) = 3$.

$t = 0, x = 3$

$$\frac{3-2}{3+1} \stackrel{?}{=} \frac{1}{4}e^{3\cdot 0}$$

$$\frac{1}{4} = \frac{1}{4}e^{0}$$

$$\frac{1}{4} = \frac{1}{4}\cdot 1$$

$$\frac{1}{4} = \frac{1}{4} \quad \text{yes!}$$

The solution satisfies initial condition

$$\frac{d}{dt}\left(\frac{x-2}{x+1}\right) = \frac{d}{dt}\left(\frac{1}{4}e^{3t}\right)$$

Chain Rule: $\underbrace{\frac{d}{dt} f(x(t)) = \frac{df}{dx}\cdot\frac{dx}{dt}}_{\text{Left Hand Side}}$

$$\left[\frac{(x+1)\cdot 1 - (x-2)\cdot 1}{(x+1)^2}\right]\frac{dx}{dt} = \frac{3}{4}e^{3t}$$

$$\frac{x+1-x+2}{(x+1)^2}\frac{dx}{dt} \stackrel{?}{=} \frac{3}{4}e^{3t}$$

$$\frac{3}{(x+1)^2}\frac{dx}{dt} = \frac{3}{4}e^{3t}$$

$$\frac{1}{(x+1)^2}\frac{dx}{dt} = \frac{1}{4}e^{3t}$$

$$\frac{dx}{dt} = (x+1)^2\frac{1}{4}e^{3t}$$

$$= (x+1)^2\frac{(x-2)}{(x+1)}$$

$$\frac{dx}{dt} = (x+1)(x-2) \quad \text{yes!}$$

Solution satisfies rate eqn