Math 114 Fall 2005 **BONUS** Quiz 4

SHOW ALL YOUR WORK

1. *(10 points total)* The number n of neurons in a human brain is related to the alcohol level a (both of these are function of age t) by the equation

$$na^2 + n^2 e^a = \text{constant}.$$

If at a certain age, someone has an alcohol level of zero but is increasing their alcohol level at a rate of 2 grams per month, what is the *relative* rate of change of the number of neurons in their brain?

NOTE: the *relative* rate of change of n is defined to be $\frac{1}{n}\frac{dn}{dt}$.

Differentiate the equation with respect to time, i.e. $\frac{d}{dt}$

$$\frac{d}{dt}[na^2 + n^2 e^a] = \frac{d}{dt}[\text{constant}]$$

We know n is a function of time, $n(t)$ and a is a function of time $a(t)$. Using the Product & Chain Rules

$$\frac{dn}{dt}\cdot a^2 + n\cdot 2a\cdot\frac{da}{dt} + 2n\frac{dn}{dt}\cdot e^a + n^2\cdot e^a\frac{da}{dt} = 0$$

$$\frac{dn}{dt}[a^2 + 2ne^a] + \frac{da}{dt}[2an + n^2e^a] = 0$$

$$\frac{dn}{dt} = \frac{-\frac{da}{dt}[2an + n^2 e^a]}{a^2 + 2ne^a}$$

$$\frac{1}{n}\frac{dn}{dt} = \frac{-\frac{da}{dt}[2an + n^2e^a]\cdot\frac{1}{n}}{a^2 + 2ne^a} = -\frac{da}{dt}\frac{[2a + ne^a]}{a^2 + 2ne^a}$$

We know at a certain time $a(t^*) = 0$ $\frac{da}{dt} = 2$ g/month

$$\frac{1}{n}\frac{dn}{dt} = -2\cdot\frac{[2\cdot 0 + n\cdot 1]}{[0^2 + 2n\cdot 1]}$$

$$= -2\cdot\frac{n}{2n} = -1$$

$$\boxed{\frac{1}{n}\frac{dn}{dt} = -1 \text{ g/month}}$$