Math 114 Fall 2005 Quiz **TWELVE**

Show all your work and explain all your answers.

1. Consider the function $z = f(x,y) = x^3 + y^3 - 6xy + 1$ on the unconstrained domain of **all** (x,y) values in the xy-plane.

a. *(2 points)* Compute $f_x(x,y)$ and $f_y(x,y)$.

$f_x = 3x^2 - 6y$

$f_y = 3y^2 - 6x$

b. *(6 points)* A **critical point** of a function $f(x,y)$ is a point (a,b) in the domain of f for which both the equations $f_x(a,b) = 0$ and $f_y(a,b) = 0$ are satisfied simultaneously. Find the location of all the critical points of $f(x,y) = x^3 + y^3 - 6xy + 1$.

$3a^2 - 6b = 0 \Leftrightarrow a^2 - 2b = 0$

$3b^2 - 6a = 0 \Leftrightarrow b^2 - 2a = 0$

$a^2 = 2b \Rightarrow b = \frac{a^2}{2}$

$b^2 = 2a$

$\left(\frac{a^2}{2}\right)^2 = 2a$

$\frac{a^4}{4} = 2a$

$a^4 = 8a$

$a^4 - 8a = 0$

$a(a^3 - 8) = 0$

$a = 0$ or $a^3 - 8 = 0$

$a^3 = 8 \Rightarrow a = 2$

If $a = 0$, $b^2 = 0$ $\Rightarrow b = 0$

If $a = 2$, $b^2 = 4$

$b = \pm 2$

However if $b = -2$ then $a^2 = -4$ which is impossible

Only solutions are $(0,0)$ and $(2,2)$

b. *(2 points)* It turns out that $f(x,y) = x^3 + y^3 - 6xy + 1$ has one **local minimum** and one **saddle point** (i.e. a special kind of critical point which is neither a local maximum or local minimum of $f(x,y)$). Does $f(x,y)$ have a **global maximum value** and a **global minimum value**? CAREFULLY EXPLAIN THE REASONS FOR YOUR ANSWER ABOUT THE EXISTENCE OR NON-EXISTENCE OF GLOBAL EXTREMA FOR THIS FUNCTION.

NOTE: question doesn't ask which is saddle and which is extremum.

$f(0,0) = 0^3 + 0^3 - 6 \cdot 0 \cdot 0 + 1 = 1$

$f(2,2) = 2^3 + 2^3 - 6 \cdot 2 \cdot 2 + 1 = 8 + 8 - 24 + 1 = -7$

Clearly $f(x,y)$ can NOT have a GLOBAL MAX since it does not have a LOCAL MAX.

Clearly $f(x,y)$ does NOT have a GLOBAL MIN because by choosing large negative values of x and y causes $f(x,y) \to -\infty$ so there is no largest negative output for $f(x,y)$.