SHOW ALL YOUR WORK

1. *(10 points total)* **Question 1, Exam 1, Fall 1998.** Suppose we are trying to model a disease, with the following assumptions:

(i) All infected people recover in ten days. After recovery, you will always have immunity, and will never become susceptible again.

(ii) The total population does not change, i.e. there are no births or deaths.

(iii) The transmission coefficient, a, is .00002 .

(iv) Right now there are 3200 people who have not had the disease, 2000 who currently have it and 850 who have had it already.

(a) *(3 points)* Write down a system of initial value problems which represent the S-I-R model for this disease, using the above assumptions.

$$S' = -0.00002SI$$
$$I' = 0.00002SI - \frac{1}{10}I$$
$$R' = \frac{1}{10}I$$

(b) *(2 points)* What is the threshold value for this model?

$$S_* = \frac{b}{a} = \frac{1/10}{0.00002} = \frac{1}{0.0002} = 5000$$

(c) *(3 points)* Calculate how many people per day are currently getting infected right now. (BE CAREFUL! You should try reading this question *twice* before answering it.)

Number of people getting infected per day
= Number of people LEAVING the susceptible population

$a\,S(0)I(0) = -(0.00002)(3200)(2000) = -128$ people per day

(d) *(2 points)* As time goes on, will the number of susceptible people decrease at a faster and faster rate, or more and more slowly? **Explain why.** (BE CAREFUL! Think about exactly what mathematical quantity related to S this question is asking about or perhaps try to visualize the graph of S to assist you in answering this question.)

This question is asking about S'', the second derivative of S or the concavity of the graph of S. Since $S' \propto S$ and $S' \propto I$ and we know $S(0) < S_*$ then S and I will both decrease, so S' will also decrease, so the rate at which S decreases, will decrease. S will remain concave up.