

NAMES: _____

Lab 8: Visualizing Functions of Two Variables

Objectives:

1. To be introduced to tools which can be used to graphically represent functions of two variables
2. To obtain experience with the graphical behavior of surfaces
3. To become familiar with both vertical and horizontal slices of multivariable functions

Introduction

The purpose of this lab is to explore, graphically and numerically, functions of two variables. We will do this by using several different techniques to gain information about surfaces determined by functions of two variables.

§1 Vertical Slices

One approach is to view the graph in *slices*. Previously in class we have defined vertical slices.

DEFINITION: (Vertical Slice)

- Suppose $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, $z = f(x, y)$ is a function of two variables.

The **vertical slice of f along the x -axis, holding $y = b$** is the function

$$g(x) = f(x, b), \quad \text{provided } (x, b) \in U.$$

Similarly, the **vertical slice of f along the y -axis, holding $x = a$** is the function

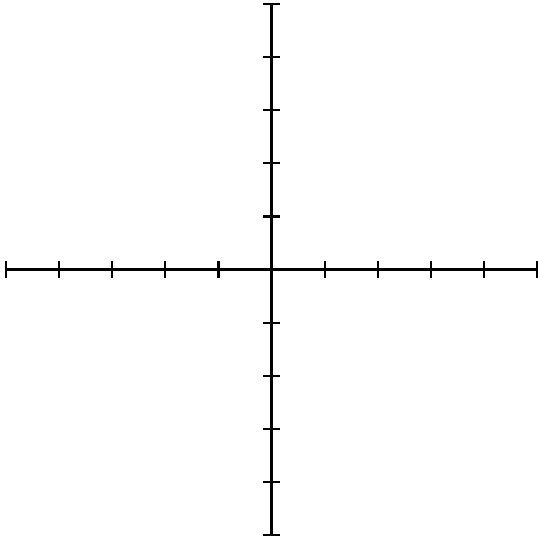
$$\phi(y) = f(a, y), \quad \text{provided } (a, y) \in U.$$

On the next few pages, sketch vertical slices for the following functions letting $a = -2, -1, 0, 1$ and 2 and $b = -2, -1, 0, 1$ and 2 .

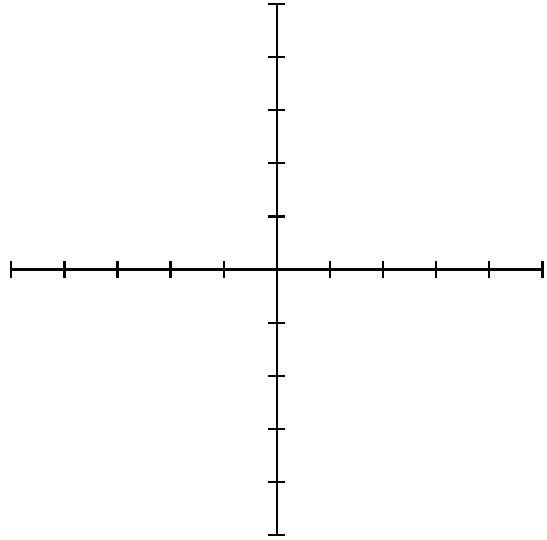
VERTICAL SLICES

$$g(x, y) = x^2 + y^2$$

Along x -axis

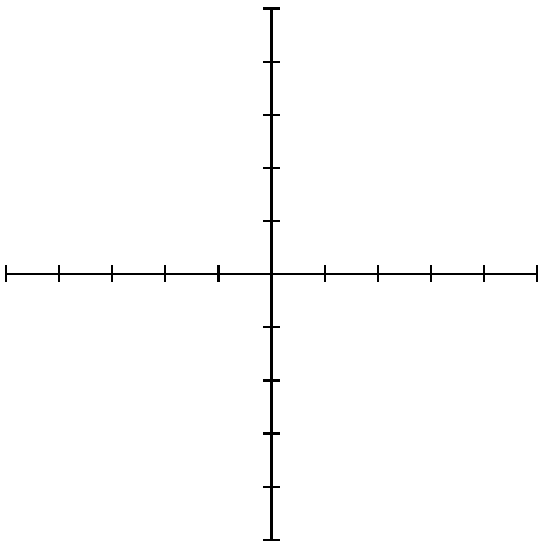


Along y -axis

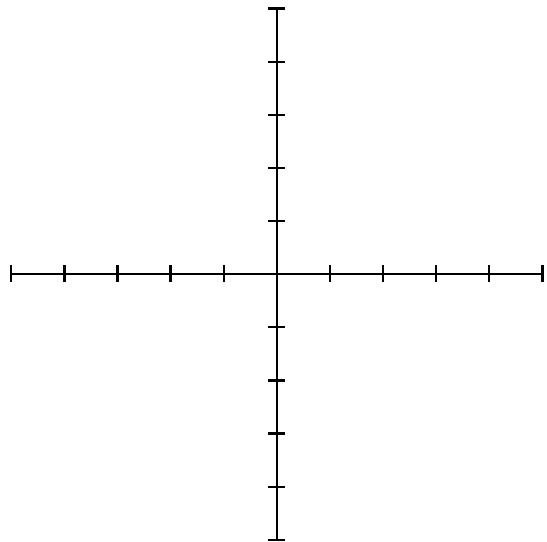


$$j(x, y) = x^2 - y^2$$

Along x -axis



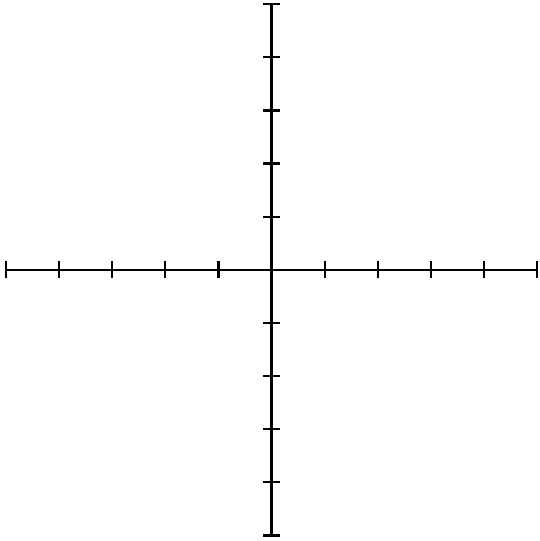
Along y -axis



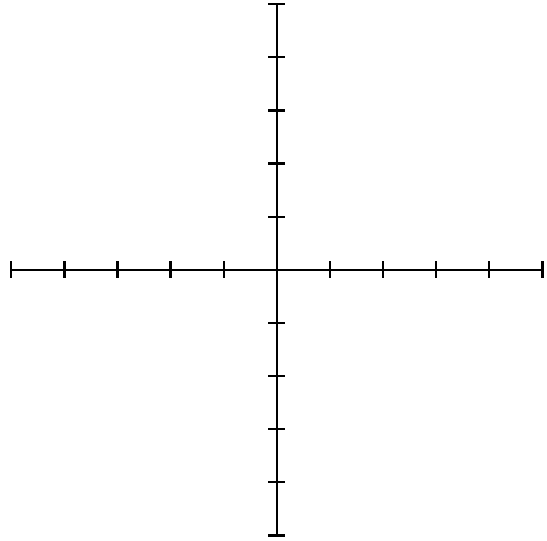
VERTICAL SLICES

$$f(x, y) = (x + y)^2$$

Along x -axis

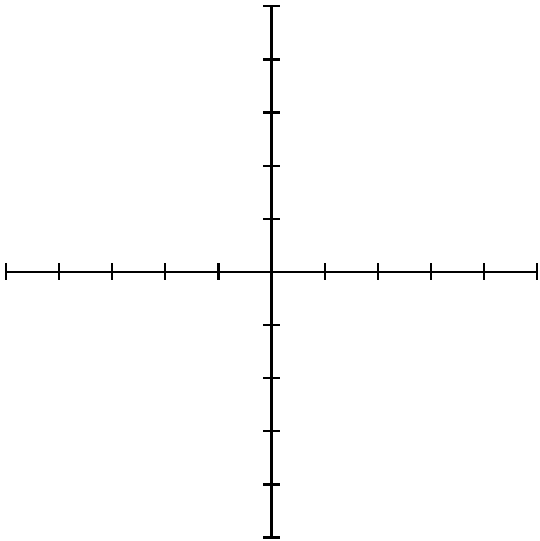


Along y -axis

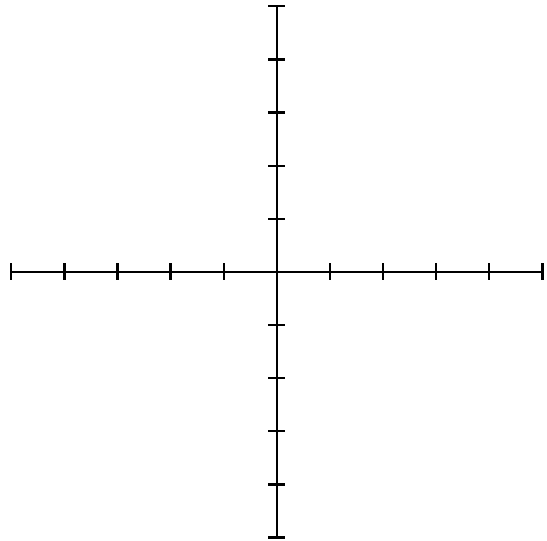


$$k(x, y) = \sin(x) - \sin(y)$$

Along x -axis



Along y -axis



§2 Horizontal Slices

It requires mathematical perception and artistic ability to sketch graphs of functions of two variables in a meaningful way. Another alternative to obtain visual information about a multivariable function is to use **vertical slices**. That is one plots the level curves (or contour lines) for the function on a two dimensional graph.

DEFINITION (Contour or Level Set):

Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$, $z = f(x, y)$ is a function of two variables.

A **contour (or level set)** of f is the set of points for which the function is constant, i.e. $z = f(x, y) = C$. One can almost think of a contour as a **horizontal** slice of the surface $z = f(x, y)$ at the value $z = C$. Note, contours (like vertical slices) can also be graphed in a 2-D plane.

Let us examine $g(x, y) = x^2 + y^2$. To find the level curve for $C = 1$, we set $g(x, y)$ equal to 1 and plot the points (x, y) which satisfy the equation

$$x^2 + y^2 = 1.$$

If we do the same for $C = 0$ and $C = 4$ and 9, we can get a good idea of the general shape of the three-dimensional graph. Sketch your contours on the CONTOUR PLOTS page following. Then do the same for:

- a. $j(x, y)$; $C = -4, -2, 0, 2, 4$
- b. $f(x, y)$; $C = 0, 1, 4, 9$
- c. $k(x, y)$; $C = -2, 0, 2$

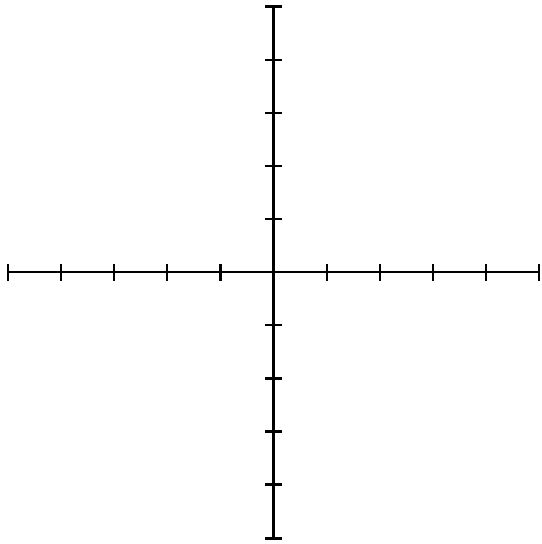
Finding the level curves for $k(x, y)$ for constants other than the ones given ($k = -2$, $k = 0$ and $k = 2$) is a bit more involved. There are numerous software programs we can use to help us.

Let's use the program **WINPLOT**. If Winplot is not on the computer you are working on you can download it by going to <http://www.google.com> and searching on "Winplot." It is also directly available at <http://math.exeter.edu/rparris/winplot.html> The file downloads very quickly as an executable file. I would recommend saving it in your personal network drive. Contour diagrams consist of several implicitly defined two-dimensional curves so select the "Window-*i*2-dim" option or hit the key **F2**. Then select "Implicit..."

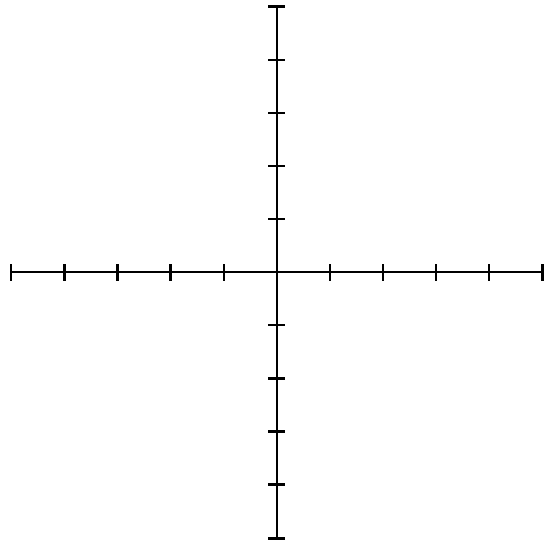
Your task is to have Winplot graph the contours for *any* level curve $f(x, y) = c$ for your multivariable function $f(x, y)$. Put the equation of your function in the highlighted box as $xx + yy = c$ and click ok. Then select the "family" button and select a range of values for c . Another way to look at the family of curves is to use the "Anim" tab. Make sure you select the same letter "c" that you used when you implicitly defined the family of curves.

CONTOUR PLOTS (LEVEL CURVES)

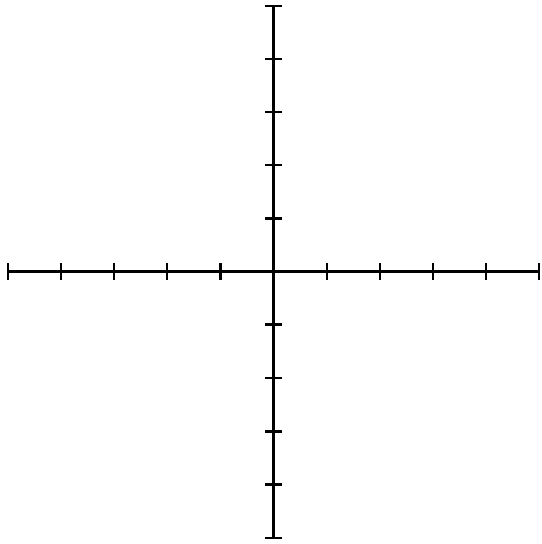
$$g(x, y) = x^2 + y^2$$



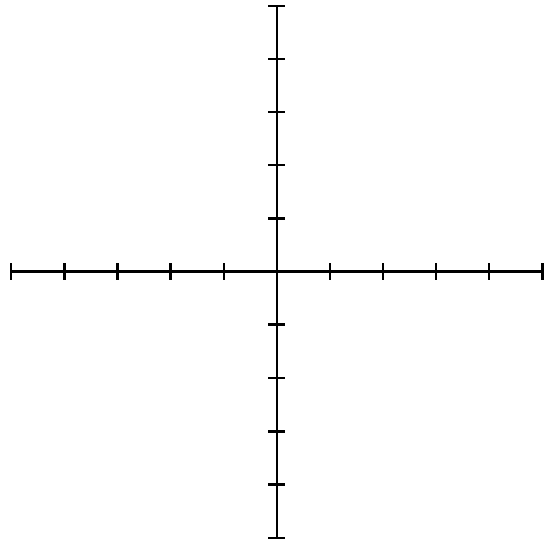
$$j(x, y) = x^2 - y^2$$



$$f(x, y) = (x + y)^2$$



$$k(x, y) = \sin(x) - \sin(y)$$



§3 Surface Plots

Our goal is to be able to produce surface plots of a function of two variables $z = f(x, y)$. This will be a 2-D representation of how three variables x, y and z are varying with respect to each other. As you might guess, technology is a very useful tool in this endeavor.

Using DERIVE

DERIVE can be used to give you a wireframe picture of the graph of a function in two variables. Use the author command to enter your function. (Don't forget the `:=`) Then click on the 3-d graphics button. Tile your windows, and then click graphics window and on the 3-d button.

Using MATLAB

Matlab can also be used to generate surface plots also. The following commands will produce a plot of the function $z = f(x, y) = 2 - x^2 + y^2$ on $-5 \leq x \leq 5, -5 \leq y \leq 5$ with contours beneath it. **NOTE: you need to type in the commands very precisely.** You will of course have to change the third line with your given function. You might want to try using the Matlab command `mesh`, `surf` or `surf1` instead of `surfc` on the last line.

```
x=linspace(-5,5,20);
y=linspace(-5,5,20);
[X,Y]=meshgrid(x,y);
Z=2 - X.^2 + Y.^2;
surfc(x,x,Z);
```

Using MATHEMATICA

Mathematica can also be used to help you visualize surface plots. You have to press SHIFT-ENTER in order to execute a command. This first command will plot the surface plot of $f(x, y) = 2 - x^2 + y^2$

```
Plot3D[2-x^2+y^2, {x,-5,5},{y,-5,5}]
```

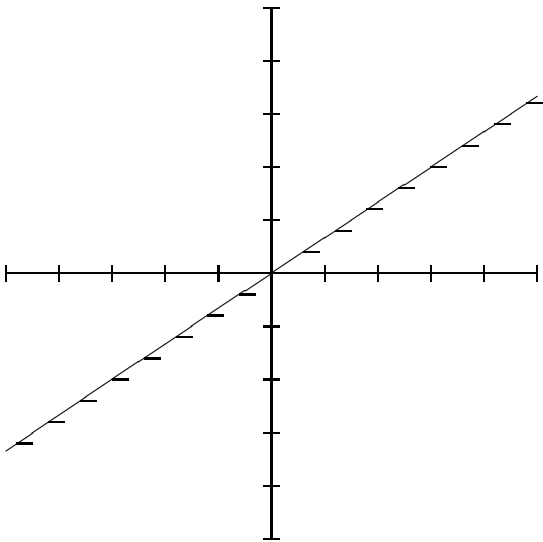
This next command will plot the contours of $f(x, y) = 2 - x^2 + y^2$

```
ContourPlot[2 - x^2 + y^2, {x, -5, 5}, {y, -5, 5}]
```

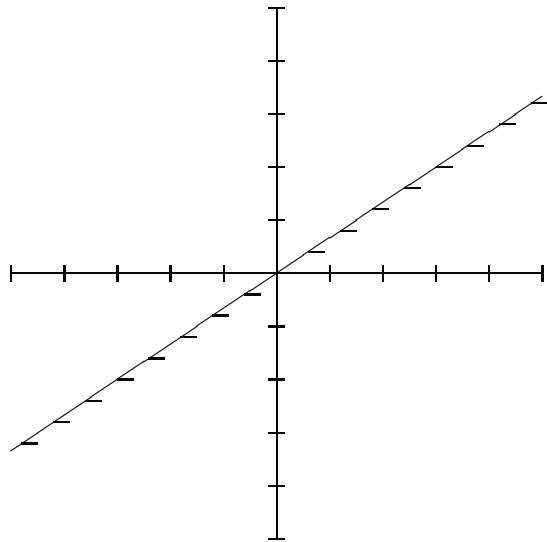
Mathematica and Matlab are two powerful computer programs which can be used as tools by mathematicians to solve very complicated problems.

3-D SURFACE PLOTS

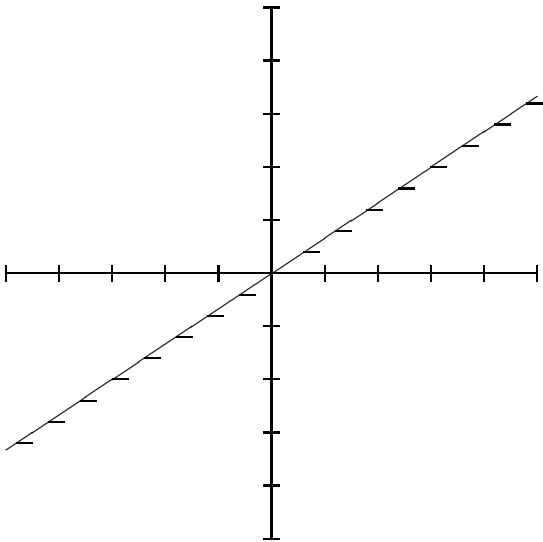
$$g(x, y) = x^2 + y^2$$



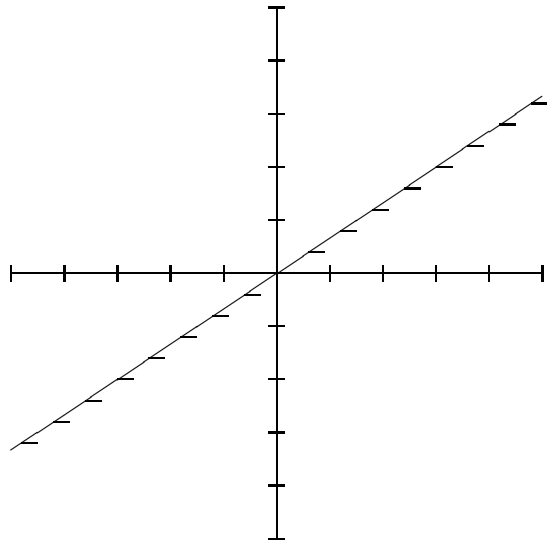
$$j(x, y) = x^2 - y^2$$



$$f(x, y) = (x + y)^2$$



$$k(x, y) = \sin(x) - \sin(y)$$



§4 Assignment

Write-up: Each team must turn in one copy of this worksheet, completely filled out. Your sketches should approach (in the limit) works of art. **In particular**, draw the contour map, draw *multiple* vertical slices along the x and y axes, and hand in **Monday/Tuesday December 5/6**.