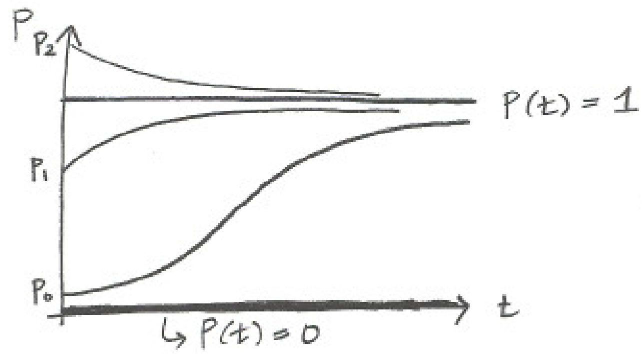
2. Logistic Model: $3P - 3P^2$

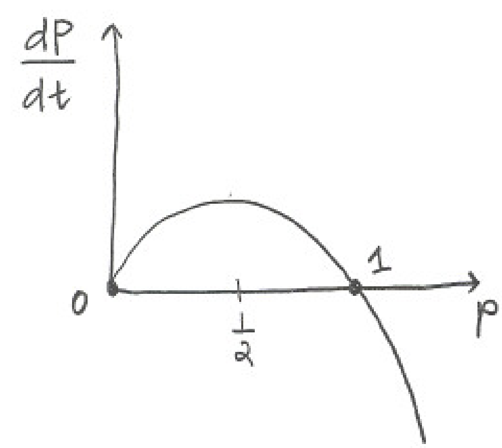
(a) Such solution curves look like



(b) A stable value of the population is $P(t) = 1$.

(c) $P(t)$ stays constant if $P_0 = 0$ or $P_0 = 1$. If $0 < P_0 < 1$, $P(t)$ increases until it is near 1. If $P_0 > 1$, $P(t)$ decreases until it is near 1. In the long term, any non-zero solutions approach 1. Note that when $0 < P_0 < 1$ and $P(t)$ is increasing, there is an inflection point at $P^* = \frac{1}{2}$; hence, solutions that start below $\frac{1}{2}$ will change concavity.

(d)



$\frac{dP}{dt}$ is positive on $(0,1)$ which corresponds to increasing solutions $P(t)$. It has a max at $P = \frac{1}{2}$ which corresponds to the inflectio point. $\frac{dP}{dt}$ crosses the P-axis at $P=0$ and $P=1$; these are the equil brium solutions. $\frac{dP}{dt} < 0$ for all $P > 1$ The slope function has no max or min there so there is no correspond inflection pt. in the solutions.

6. $M :=$ constant population size

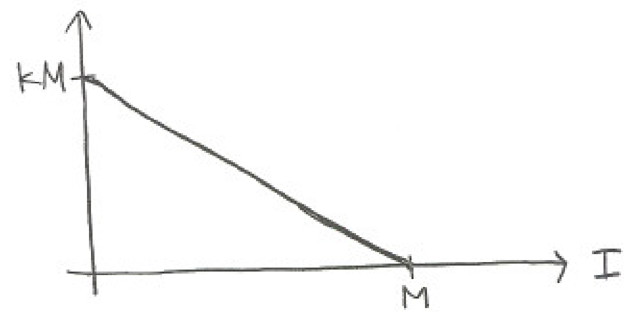
(a) Let $I :=$ informed people, $\mathcal{U} :=$ uninformed people.

Then: $I + \mathcal{U} = M$ and $\mathcal{U} = M - I$.

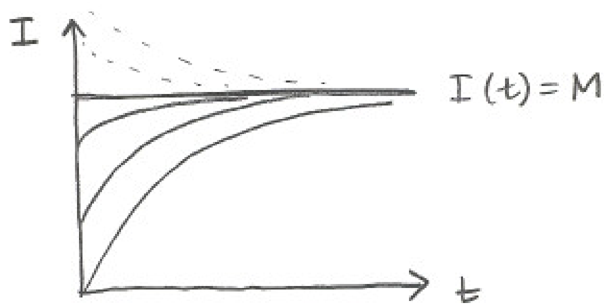
We have $\frac{dI}{dt} \propto \mathcal{U} \Rightarrow \boxed{\frac{dI}{dt} = k\mathcal{U} = k(M-I)}$

i.e: $\frac{dI}{dt} = k(M-I)$

The slope function looks like:



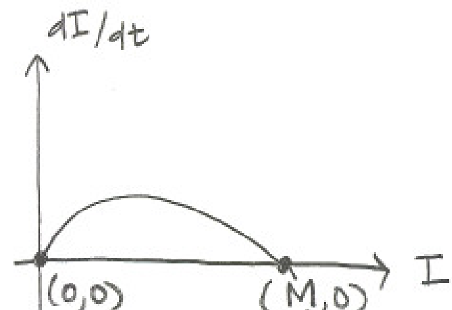
So, solution curves look like:



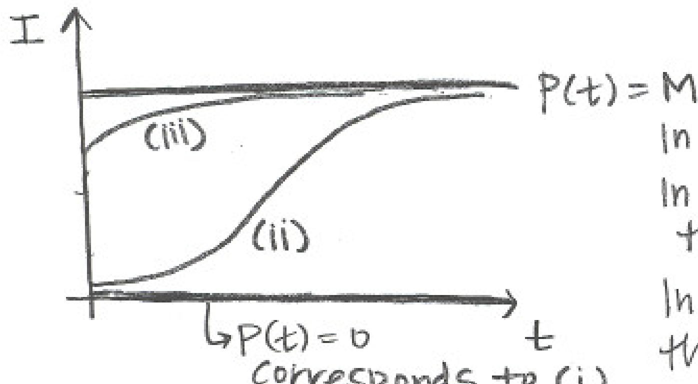
(b) In this case $\frac{dI}{dt} \propto I \cdot \mathcal{U} \Rightarrow \frac{dI}{dt} = kI\mathcal{U} = kI(M-I)$

or $\boxed{\frac{dI}{dt} = kMI - kI^2}$

Now our slope function looks like:



So solution curves look like:



In (i) there is no "fastest" spread

In (ii) the spread is fastest at the inflection point, $M/2$.

In (iii) the spread is fastest at the very beginning $(t=0)$.

7\. In this case, use the relative birth and death rates to determine the relative growth rate.

Assuming relative growth is a linear function of P:

$$\frac{1}{P}\frac{dP}{dt} = a + bP$$

So when $P = 600$:

rel. growth = rel. births − rel. deaths = $a + b(600)$:

$$.35 - .15 = a + b(600)$$

$$.2 = a + 600b$$

when $P = 800$: $.3 - .2 = a + b(800)$

$$.1 = a + 800b$$

To get the differential equation, solve the system:

$.2 = a + 600b$
$.1 = a + 800b$ $\Rightarrow$ $.1 = -200b$
$b = -.0005$

Substituting: $.2 = a + 600(-.0005) \Rightarrow a = .5$

So the d.e for the elk, E, : $\frac{1}{E}\frac{dE}{dt} = .5 - .0005E$

or $\boxed{\frac{dE}{dt} = .5E - .0005E^2}$

(b) The equilibrium size (that is not trivial) can be found by solving $0 = .5E^* - .0005E^{*2} = E^*(.5 - .0005E^*)$

or $.5 - .0005E^* = 0 \Rightarrow \boxed{E^* = 1000}$

If the population is currently 900, the population should continue to slowly increase.

This problem cont'd next page

7 cont'd:

(c) Adding 450 elk to the island pushes the population above the carrying capacity. Hence the population will start decreasing until it nears the equilibrium value.

(d)

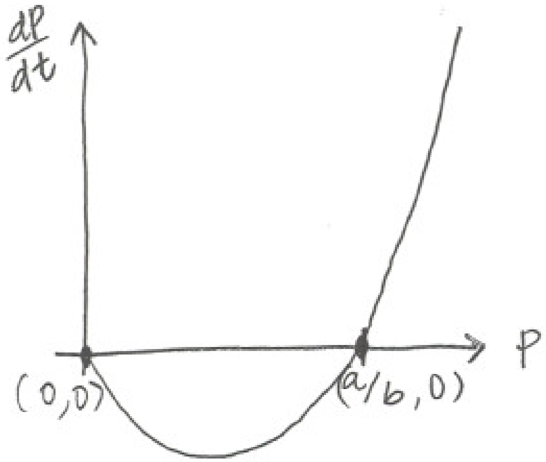
13. $f(x) = x^3$

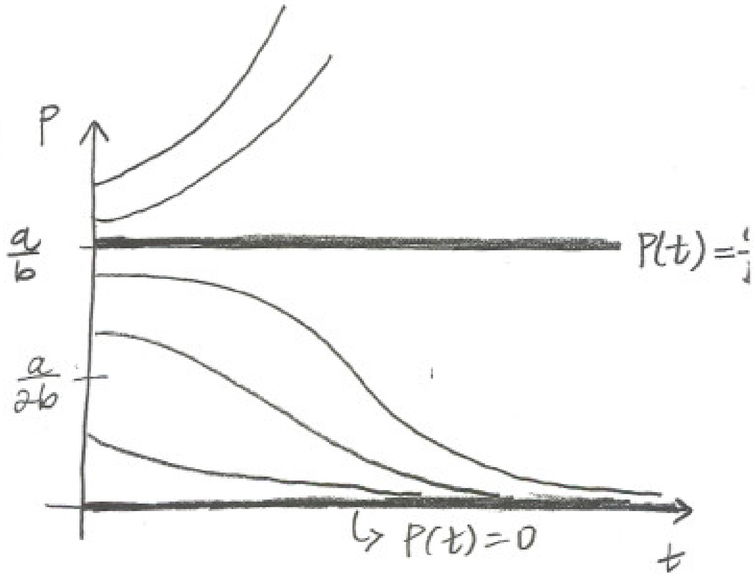
h	one sided: $(f(2+h)-f(2))/h$	two sided: $(f(2+h)-f(2-h))/2h$
.1	$\frac{(2+.1)^3-(2)^3}{.1} = 12.61$	$\frac{(2+.1)^3-(2-.1)^3}{(2\cdot.1)} = 12.01$
.01	$\frac{(2+.01)^3-(2)^3}{.01} = 12.0601$	$\frac{(2+.01)-(2-.01)^3}{(2\cdot.01)} = 12.0001$
.001	$\frac{(2+.001)^3-2^3}{.001} = 12.006001$	$\frac{(2+.001)^3-(2-.001)^3}{(2\cdot.001)} = 12.000001$

In both cases the error decreases with h (the derivative $f'(2) = 12$) in the 1-sided case the error is $\leq h\cdot(6.1)$. In the 2 sided case the error equals h^2. (In each case the 2-sided approximation gives a better estimate)

19. $\dfrac{dP}{dt} = aP^2 - bP \qquad a,b>0$

$P(a-bP)$

i) 

(b) 

From our graph of the slope function, we can tell that P decreases between 0 and a/b with an inflection point at $\frac{a}{2b}$. P increases (without a concavity change) for values above $\frac{a}{b}$. The equilibrium solutions correspond to the P-axis interceptions; $P^* = 0$, $P^* = a/b$.

(c) $\frac{a}{b}$ is called a threshold population since, above $\frac{a}{b}$, the population grows and below $\frac{a}{b}$ the population dies out. In other words $\frac{a}{b}$ is the minimum initial population that will allow the population to survive.