HOMEWORK 8 SOLUTIONS

SM 6.2: 1, 27, 30 ; 6.8 . 6, 12 ; 1.4 : 18, 26

3.1 : 1, 4, 42, 48 , 50 ; 7.6 : 16, 34, 40, 44, 48

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Section 6.2 : 27, 30

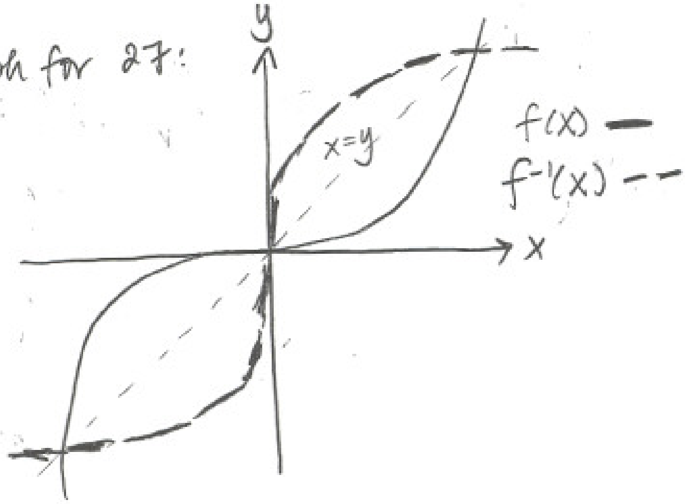
27: $x^5 + 3x^3 + x = f(x) \quad a = 5$

If $x^5 + 3x^3 + x = 5$, $x = 1$ $\quad (f^{-1}(5) = 1)$

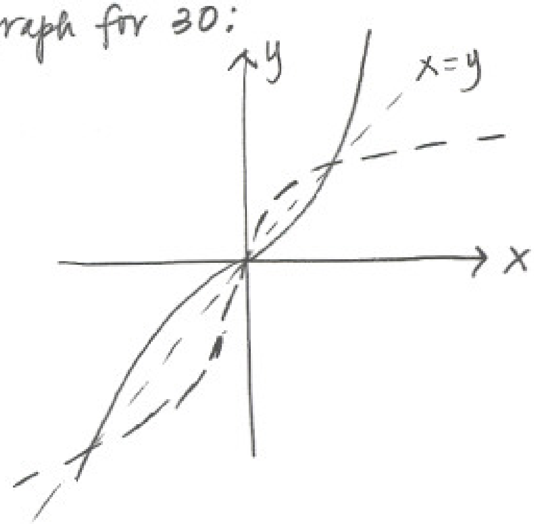
$f'(x) = 5x^4 + 9x^2 + 1 \qquad f'(1) = 5 + 9 + 1 = 15$

$$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))} \Rightarrow (f^{-1}(5))' = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(1)} = \boxed{\frac{1}{15}}$$

graph for 27:




graph for 30:




30: $f(x) = \sqrt{x^5 + 4x^3 + 3x + 1} \quad a = 3$

If $\sqrt{x^5 + 4x^3 + 3x + 1} = 3$, $x = 1$ $\quad (f^{-1}(3) = 1)$

$f'(x) = \frac{1}{2}(x^5 + 4x^3 + 3x + 1)^{-1/2}(5x^4 + 12x + 3)$

$$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))} \Rightarrow (f^{-1}(3))' = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \frac{1}{2} \cdot \frac{19}{\sqrt{9}} = \boxed{\frac{19}{2\sqrt{9}}}$$

Section 6.8: 6, 12 

6. $\cos^{-1}(x^3+1)$

$$[\cos^{-1}(x^3+1)]' = \frac{-1}{\sqrt{1-(x^3+1)^2}} \cdot 3x^2$$ arccos, chain rule

$$= \frac{-3x^2}{\sqrt{1-x^6-2x^3-1}} = \frac{-3x^2}{\sqrt{-x^6-2x^3}}$$ simplifying

* this has no real solution

12. $[\sin x \cdot \sin^{-1}(2x)]' = \sin x \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 + \cos x \sin^{-1}(2x)$

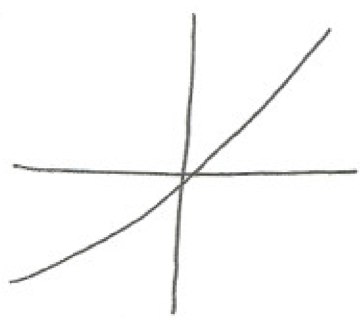
arcsin, product, chain rule

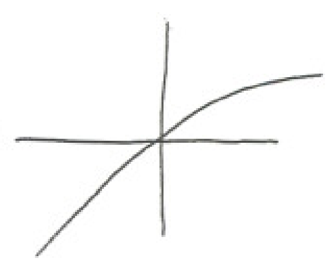
$$= \frac{2\sin x}{\sqrt{1-4x^2}} + \cos x \sin^{-1}(2x)$$

simplifying

6.2: 1 (I forgot it on previous page)

If $f'(x) > 0$, $f(x)$ is always increasing. The graphs at the right are 2 such examples. They will always pass the horizontal line test since in order to fail the test, $f'(x)$ would have to change signs so that there would be $x_1$, $x_2$ with $f(x_1) = f(x_2)$.





3.1: 4, 42, 48, 50

③

4: Since $f(t)$ and $g(t)$ represent runner position, we can assume that $f(0) = g(0) = 0$, or, in other words, that the runner is at the "start." Initially, one runner is going twice as fast as the other; near zero, $f'(t) = 2g'(t)$ (because velocity is the first derivative of position). Considering $\lim_{t \to 0^+} \frac{f(t)}{g(t)}$ it is indeterminate so L'Hôpitals Rule can be appleied then $\lim_{t \to 0^+} \frac{f(t)}{g(t)} = \lim_{t \to 0^+} \frac{f'(t)}{g'(t)}$

$$= \lim_{t \to 0^+} \frac{2g'(t)}{g'(t)} = \lim_{t \to 0^+} 2 = 2.$$

42: $\lim_{x \to 0} \frac{\sin(\sin x)}{\sin x}$ check: $\frac{0}{0}$ indeterminate

$$\overset{L'H}{=} \lim_{x \to 0} \frac{\cos(\sin x) \cdot \cos x}{\cos x} = \lim_{x \to 0} \cos(\sin x) = \cos 0 = \boxed{1}$$

48. $\lim_{x \to 0} \frac{\sin x}{x^2} = \lim_{x \to 0} \frac{\cos x}{2x}$

There are no errors in this step. The initial limit is indeterminate; L'Hôpitals Rule is applied and the derivatives are correct.

$\lim_{x \to 0} \frac{\cos x}{2x} = \lim_{x \to 0} \frac{-\sin x}{2}$

This is where an error occurs. The $\lim_{x \to 0} \frac{\cos x}{2x}$ is not indeterminate so L'Hôpitals Rule cannot be appleied. The limit should have been determined as $\infty$.

50. $\lim_{x \to 0} \frac{\sin nx}{\sin mx}$ This limit is indeterminate $0/0$ (assuming $n \neq m$), so L'Hôpitals rule can be appeled.

$\stackrel{L'H}{=} \lim_{x \to 0} \frac{n \cdot \cos nx}{m \cdot \cos mx} = \boxed{\frac{n}{m}}$ Since both $\lim_{x \to 0} \cos nx$, $\lim_{x \to 0} \cos mx =$.

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Section 7.6: 16, 34, 40, 44, 48

16. $\lim_{x \to 1} \frac{\ln x}{x-1}$ This limit is indeterminate $\frac{0}{0}$ so L'Hôpital can be appeled.

$\stackrel{L'H}{=} \lim_{x \to 1} \frac{1/x}{1} = \frac{1}{1} = \boxed{1}$

34. $\lim_{x \to \infty} (\ln x - x)$ This limit is indeterminate $\infty - \infty$; it needs to be rewritten before L'Hôpital can be appeled.

Let $y = \ln x - x \Rightarrow e^y = e^{\ln x - x} = e^{\ln x} e^{-x} = \frac{e^{\ln x}}{e^x} = \frac{x}{e^x}$
$(\ln e^y = y \ln e = y)$

consider $\lim_{x \to \infty} e^y = \lim_{x \to \infty} \frac{x}{e^x}$ (indeterminate $\frac{\infty}{\infty}$, apply L'H)

$\stackrel{L'H}{=} \lim_{x \to \infty} \frac{1}{e^x} = 0$

Therefore $\lim_{x \to \infty} y = \lim_{x \to \infty} \ln e^y = \ln\left(\lim_{x \to \infty} \frac{1}{e^x}\right) = \boxed{-\infty}$

40. $\lim_{x \to 0^+} (\cos x)^{\frac{1}{x}}$

indeterminate $1^\infty$.
We did this in class ☺ so
I won't rewrite it here.

$\lim_{x \to 0^+} (\cos x)^{\frac{1}{x}} = 1$

44. $\lim_{x \to 0} \frac{e^x - 1}{x^2} = \lim_{x \to 0} \frac{e^x}{2x}$

This step is correct: since the first limit is indeterminate, apply L'H. Derivatives are correct.

$\lim_{x \to 0} \frac{e^x}{2x} = \lim_{x \to 0} \frac{e^x}{2}$

This step is incorrect since the $\lim_{x \to 0} \frac{e^x}{2x}$ is NOT indeterminate.

48. * your text uses d.n.e. as $\infty$ in this case.

(a) $\lim_{x \to \infty} e^{x^2} - e^x$

(b) $\lim_{x \to \infty} e^{x+1} - e^x$

(c) $\lim_{x \to \infty} \ln(e^2 x) - \ln x$

1.4 : 18, 26

18. $\lim\limits_{x\to\infty} \dfrac{-x}{\sqrt{4+x^2}}$ indeterminate $\left(\frac{-\infty}{\infty}\right)$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{-1}{\frac{1}{2}(4+x^2)^{-1/2}\cdot 2x} = \lim_{x\to\infty} \frac{-2\sqrt{4+x^2}}{2x}$$

$$= \lim_{x\to\infty} -2\sqrt{\frac{4+x^2}{4x^2}} = \lim_{x\to\infty} -2\sqrt{\frac{1}{x^2}+\frac{1}{4}}$$

$$= -2\sqrt{\frac{1}{4}} = -2\cdot\frac{1}{2} = \boxed{-1}$$

26. $\lim\limits_{x\to-\infty} \dfrac{x+\cos x}{3x+2}$ indeterminate $\left(\frac{-\infty}{-\infty}\right)$

$$\overset{L'H}{=} \lim_{x\to-\infty} \frac{1-\sin x}{3} = \lim_{x\to-\infty} \frac{1}{3} - \frac{\sin x}{3}$$

$$= \lim_{x\to-\infty} \frac{1}{3} - \lim_{x\to-\infty} \frac{\sin x}{3}$$

But $\lim\limits_{x\to-\infty} \dfrac{\sin x}{3}$ does not exist.

$$\boxed{\text{So } \lim_{x\to-\infty} \frac{x+\cos x}{3x+2} \text{ d.n.e.}}$$