Class 29

Monday November 16

Limits of Functions at Undefined x-values and L'Hôpital's Rule

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L'Hôpital's Rule

If the limit on the left has an indeterminate form (i.e. $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$ or $\pm\infty \cdot 0$) then it is equal to the limit on the right (if this limit exists)

$$\lim_{x\to b} \frac{f(x)}{g(x)} = \frac{\lim_{x\to b} f'(x)}{\lim_{x\to b} g'(x)}$$

By using this new rule we can find the limits of a whole bunch of new functions, and we have an easier way to find horizontal asymptotes:

Examples

Take the following limits by first identifying which indeterminate form they take and then apply L'Hopital's Rule.

1. $\lim_{x\to\infty} \dfrac{5+5x}{3x-2}$

2. $\lim_{x\to-\infty} \dfrac{5+5x}{3x-2}$

3. $\lim_{x\to 1}(x-1)^3 \ln(x-1)$

4. $\lim_{x\to 0} \dfrac{\cos(x) - 1 + \frac{1}{2}x^2}{\sin(x) - x}$

Limits of Functions at Undefined x-values

If a function $f(x)$ is defined for all points near an x-value a, but is undefined at a itself, we can ask ourselves what the limit of the function is as x approaches a from either values smaller than a or greater than a or both, i.e. $\lim_{x\to a^-} f(x)$ OR $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a} f(x)$ is $+\infty$ or $-\infty$
Just because the function is undefined at a does not mean the limits will be undefined.

Vertical Asymptotes

A function $f(x)$ has a vertical asymptote at $x = a$ if any of the three limits $\lim_{x\to a^-} f(x)$ OR $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a} f(x)$ is $+\infty$ or $-\infty$

Examples:

For each of the functions below, determine for which x values the function is undefined and thus find out if the function has any **vertical asymptotes** at these points by taking the limit of the function at this point (or points).
(If you have extra time, you should find the **horizontal** asymptotes too.)

5. $f(x) = \dfrac{\sin(x)}{x}$

6. $g(x) = \dfrac{x^2 - 4}{x - 2}$

7. $k(x) = \tan(x)$

8. $m(x) = \dfrac{1}{3x - 2}$

9. $n(x) = \dfrac{5 + 5x}{3x - 2}$

10. $p(x) = \dfrac{(3x + 2)(x - 7)}{(x + 1)(4x + 3)}$

11. $l(x) = e^{\frac{1}{x}}$

ANNOUNCEMENTS

Homework: *H-H* DO **page 66 #23** and **page 232 # 10, 11, 18, 19** for **Wed Nov 18**
Reading: *H-H* READ 127-136
Reminder: Exam 3 is scheduled for **Monday November 23** in class
Reminder: GATEWAY Exams need to be passed by the end of the semester or el se your grade is **automatically** reduced. Absolutely No Exceptions.