Class 27

Wednesday November 11

Infinte Limits and Horizontal Asymptotes

We have learned how to apply our ability to differentiate functions in order to find intervals on which functions are increasing and decreasing, as well as classify local maxima and minima, determine concavity and find roots of the function. This is ALMOST enough to sketch a graph by hand of most functions. The last piece we need is the ability to deal with horizontal and vertical asymptotes.

Consider the function $f(x) = \dfrac{4x^2}{x^2+1}$ on the *infinite interval* $(-\infty, +\infty)$.

Let's sketch it below:

x	$-\infty \leftarrow$	-1000	-100	-10	-1	0	1	10	100	1000	$\rightarrow +\infty$
$f(x)$		3.999996	3.9996	3.96	2	0	2	3.96	3.9996	3.999996	

This table suggest that the value of $f(x)$ approaches the value of ______ as x increases without bound (we say $x \to \infty$)

The function $f(x)$ approaches the value of ______ as x *decreases* without bound.

Mathematically, we can write these sentences as:

$$\lim_{x\to\infty} f(x) = 4 \text{ and } \lim_{x\to-\infty} f(x) = 4$$

Thus the function $f(x)$ has horizontal asymptotes at $y = 4$.

Horizontal Asymptote

The line $y = L$ is a **horizontal asymptote** of the graph $f(x)$ if $\lim\limits_{x\to-\infty}$ or $\lim_{x\to\infty} f(x) = L$

Considering this definition a function can have AT MOST ____ asymptotes.

Some Rules Involving Limits at Infinity

If $\lim\limits_{x\to\infty} f(x) = F$ and $\lim\limits_{x\to-\infty} g(x) = G$ then

1. $\lim\limits_{x\to\infty} [f(x) + g(x)] = F + G$
2. $\lim\limits_{x\to\infty} [f(x) \cdot g(x)] = F \cdot G$
3. If r is a positive real number and c is any real number, $\lim\limits_{x\to\infty} \dfrac{c}{x^r} = 0$

<u>**Exercise**</u>

1. $\lim\limits_{x\to\infty} 7 - \dfrac{1}{\sqrt{x}} =$

2. $\lim\limits_{x\to\infty} \dfrac{x + 300000}{x - 1} =$

3. $\lim\limits_{x\to-\infty} \dfrac{x + 300000}{x - 1} =$

<u>Examples</u>

Evaluate the following limits and compare the results. Notice a pattern?

4. $\lim_{x\to\infty} \dfrac{3x-5}{4x^2+1}$

5. $\lim_{x\to\infty} \dfrac{3x^2-5}{4x^2+1}$

6. $\lim_{x\to\infty} \dfrac{3x^3-5}{4x^2+1}$

<u>Exercise</u>

7. Consider the function $g(x) = \dfrac{2x-4}{\sqrt{x^2+1}}$.

Q: How many horizontal asymptotes does it have?

A: Evaluate $\lim_{x\to-\infty} \dfrac{2x-4}{\sqrt{x^2+1}}$ and $\lim_{x\to\infty} \dfrac{2x-4}{\sqrt{x^2+1}}$

<u>Infinite Limits Involving Trigonometric Functions</u>

Consider the following limits

8. $\lim_{x\to-\infty} \sin(x)$

9. $\lim_{x\to-\infty} \dfrac{\sin(x)}{x}$

Practice, Practice, Practice

10 (a) $\lim_{x\to\infty} \dfrac{x}{x^2-1} =$

10 (b) $\lim_{x\to\infty} \dfrac{5x^2+3}{x-1} =$

10 (c) $\lim_{x\to\infty} \dfrac{2x}{x-1} + \dfrac{3x}{x+1} =$

10 (d) $\lim_{x\to\infty} x\cos\left(\dfrac{1}{x}\right)$

10 (e) $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2-x}}$

ANNOUNCEMENTS

Homework: *H-H* DO **page 134 #12** for **Fri Nov 13**

Reading: *H-H* READ 127-136

Reminder: Exam 3 is scheduled for **Monday November 23** in class

Reminder: GATEWAY Exams need to be passed by the end of the semester or else your grade is **automatically** reduced. Ansolutely No Exceptions.