Math 110 | **BASIC CALCULUS I** | Fall 1998

Basic Calculus I

Class 26

Monday November 9

Understanding Limits

We have discussed the concept of successive approximations, sequences and limits but we haven't really sat down and summarized what we know about these concepts.

Warm-up

Look at the table below. Fill in the last line of the table.

The first two columns come from taking $\lim_{x\to 1}$ of the function $f(x) = x^3 - 5x$. The next four columns come from using Newton's Method on $f(x)$ using an initial guess of $x_0 = 1$ and then using a different initial guess of $x_0 = 2$.

x	$f(x)$	n	x_n	n	p_n
0	0	0	1	0	2
0.5	-2.375	2	-1	1	2.285714286
0.9	-3.771	3	1	3	2.237639989
0.99	-3.979701	4	-1	4	2.236069633
0.999	-3.997997001	5	1	5	2.236067978
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
1		∞		∞	

Which of the above **sequences** CONVERGE (i.e. have a limit)? And what is the limit in each case?

Continuity

We have learned that the limit of $f(x)$ as x approaches c does not depend on the value of f at $x = c$. It may happen, however, that the limit is precisely $f(c)$. In such case, we say that the limit can be evaluated by **direct substitution**. That is,

$\lim_{x\to c} f(x) = f(c)$ (Substitute c for x)

Such *well behaved* functions are called ______________________________ at c

Some Basic Rules Involving Limits

Let b and c be real numbers and let n be a positive integer.

1. $\lim_{x\to c} b = b$ 2. $\lim_{x\to c} x = c$ 3. $\lim_{x\to c} x^n = c^n$

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = K$

1. Scalar multiple: $\lim_{x\to c} bf(x) = bL$
2. Sum or difference: $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x\to c}[f(x)g(x)] = LK$
4. Quotient: $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x\to c}[f(x)]^n = L^n$

Exercise

Evaluate the following limit.

$\lim_{x \to 2}(7x^2 + 3x - 5) =$

Limits of Polynomial and Rational Functions

1. If p is a polynomial function and c is a real number, then $\lim_{x \to c} p(x) = p(c)$.

2. If r is a rational function given by $r(x) = p(x)/q(x)$ and c is a real number such that $q(x) \neq 0$, then $\lim_{x \to c} r(x) = r(c) = \dfrac{p(c)}{q(c)}$.

Exercise

Find the limit: $\lim_{x \to 1} \dfrac{x^2 + x + 2}{x + 1}$

The Limit of a function Involving a Radical

Let n be a positive integer. The following limit is valid for all c if n is odd, and is valid for $c > 0$ if n is even.

$$\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$$

The Limit of a Composite Function

If f and g be functions such that $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} g(x) = f(L)$, then

$$\lim_{x \to c} f(g(x)) = f(L).$$

Exercise

Evaluate the following limits.

(a) $\lim_{x \to 0} \sqrt{x^2 + 4}$

(b) $\lim_{x \to 3} \sqrt[3]{2x^2 - 10} \cdot \cos(x)$

(c) $\lim_{x \to 0} e^{\sin(x)}$

(d) $\lim_{x \to 1} \dfrac{\sqrt{x^2 + 1}}{2 + 6x \ln(x)}$

(e) $\lim_{x \to -1} (2x + 5)^{2x}$

ANNOUNCEMENTS

Homework: *H-H* DO **page 133 1, 7, 9, 15** for **Wed Nov 11**

Reading: *H-H* READ 127-136

Reminder: Exam 3 is scheduled for **Monday November 23** in class