Basic Calculus I

Class 23

Friday 10/30/98

Implicit differentiation.

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1. (a) $\dfrac{d}{dx}\left[(x^3+4x^2)^7\right] =$

(b) $\dfrac{d}{dx}\left[(\ln x)^7\right] =$

(c) $\dfrac{d}{dx}\left[(\sin x)^7\right] =$

(d) $\dfrac{d}{dx}\left[(\arcsin x)^7\right] =$

(e) $\dfrac{d}{dx}\left[(f(x))^7\right] =$

(f) So if $y = f(x)$, then $\dfrac{d}{dx}\left[y^7\right] =$

Part (e) is chain rule. Part (f) is **implicit differentiation**.
What is the difference?

To understand the MEANING of implicit differentiation in terms of rates of change, fill in the following blanks.

$$\frac{d}{dy}\left[y^3\right] =$$

So, **at y = 2**, the rate of change of y^3 is _____.
This means increasing y by 1 unit causes y^3 to increase by _____ units.

Now, suppose y is a function of x. And suppose $\dfrac{dy}{dx} = 5$.
This means increasing x by 1 unit causes y to increase by _____ units, which in turn causes y^3 to increase by _____ units.

Implicit differentiation says exactly the same thing:

$$\frac{d}{dx}\left[y^3\right] =$$

2. (a) Solve the equation $8x^3 + 2y^5 = 1$ for x in terms of y.

(b) Now solve the same equation for y in terms of x.

(c) When $x = 29$, $y =$

When $y = 132$, $x=$

(d) Is x a function of y or is y a function of x?

$\Rightarrow$ We say the equation $8x^3 + 2y^5 = 1$ gives x *implicitly* as a function of ____, while the equation $x = (1/2)\sqrt[3]{1 - 2y^5}$ gives x _____ as a function of y.

Similarly, we say the equation $8x^3 + 2y^5 = 1$ gives y *implicitly* as a function of ____, while the equation $y =$_____ gives y explicitly as a function of x.

3. (a) Can you solve the equation $x^2 + y^3 = 8 - x + xy^5$ for y in terms of x?

(b) When $x = 0$, $y =$

(c) Surprising fact: We can find the slope of the graph at $x = 0$! (as follows)
Implicitly differentiate the above equation with respect to x, i.e., apply $\dfrac{d}{dx}$ to both sides of the equation.

Now plug in $x = 0$ and $y =$____, and then solve for $\dfrac{dy}{dx}$.

4. Find the equation of the tangent line to the graph of $\ln(xy) = 2x$ at $x = 1$.

ANNOUNCEMENTS

Homework due Monday, 11/02/98:
HH, section 4.7: 1, 5, 7, 11, 13, 18.