# Basic Calculus I

*Class 19*

Monday October 19

## Applications of Derivatives: Finding Maxima and Minima

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Now that you have learned the rules of differentiation we will be thinking about how to apply the derivative to a variety of situations.

We are already aware that the derivative gives us qualitative information about the shape of the function. You may recall some of the following concepts. Begin by filling in the necessary information below:

| **function** | **derivative** |
|---|---|
| increasing | |
| | negative |
| level (flat) | |
| | undefined |
| steep (rising or falling) | |
| | small (positive or negative) |
| straight (horizontal) | |
| straight (slanted) | |

**Critical Point** A point $(c, f(c))$ is called a *critical po int* of a function $f$ if $f'(c) = 0$ or $f'(c)$ does not exist.

**The <u>First Derivative Test</u> for finding Local Extrema** Let $(c, f(c))$ be a critical point in the domain of $f$:

> If $f'$ is negative to the left of $c$ and positive to the right of $c$, then $f$ has a local minimum at $c$.
>
> If $f'$ is positive to the left of $c$ and negative to the right of $c$, then $f$ has a local maximum at $c$.

**Global Minimum and Global Maximum** The idea of finding global extrema is very important in word problems.

> $f$ has a *global minimum* at $p$ if all values of $f$ are greater than or equal to $f(p)$.
>
> $f$ has a *global maximum* at $p$ if all values of $f$ are less than or equal to $f(p)$.

**How to find Local Extremes**

1. Determine the domain of the function and identify the end points (if any).
2. Find $f'(x)$.
3. Find all roots of $f'(x) = 0$ in the domain, and find where $f'(x)$ does not exist.
4. Use the First Derivative Test to locate any local extrema.

**How to find Global Extremes**

1. Determine Local extremes $(c, f(c))$, as above.
2. If the $Domain = [a, b]$, then check the end point values $f(a)$ and $f(b)$ and determine which value among $\{f(c), f(a), f(b)\}$ is the largest - this is the global maximum; the smallest is the global minimum.
3. If you do not have a closed interval for the domain, then use the shape of the graph to determine global extremes.

**Example 1** Find the intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

**Solution:**

| | | | |
|---|---|---|---|
| Interval | | | |
| Test value | | | |
| Sign of $f'(x)$ | | | |
| Conclusion | | | |

**Example 2.** Find the relative extrema of the function $f(x) = x^4 - 6x^2 + 7$ and draw a sketch of its graph

ANNOUNCEMENTS

**Assignment:** *H-H* page 247: **1, 2, 3**

**Derivatives Gateway** in Lab on Thursday October 23.

**Reminder**: Exam 2 is scheduled for Thursday October 30 in lab

**The FINAL EXAM in Math 110 is scheduled for Thursday December 10 6:30-9:30 pm**