Math 110 **BASIC CALCULUS I** Fall 1998

Class 18

Wednesday 10/16/98

The chain rule.

Chain Rule: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$.

Example. Let $f(x) = x^{17}$, $g(x) = x^2 + 5$.
(i) $f'(x) =$

(ii) $g'(x) =$

(iii) $f(g(x)) =$

(iv) $f'(g(x)) =$

(v) $[(x^2 + 5)^{17}]' =$

1. Let $f(x) = x^{17}$, $g(x) = \sin x$. Then

$[(\sin x)^{17}]' =$

2. Find each of the following derivatives.

(a) $[(e^x + \sin x)^{17}]' =$

(b) $[e^{x^3}]' =$

An interpretation of the Chain Rule

Suppose z is a function of y, and y is a function of x.
Suppose we're given the following:

(a) The rate of change of z with respect to y is 4.
(b) The rate of change of y with respect to x is 3.

Part (a) says: For every unit increase in y, z increases by ____ units.
Part (b) says: For every unit increase in x, y increases by ____ units.

THEREFORE, if we increase x by 1 unit, y will increase by ____ units, which in turn causes z to increase by ____ units.

Notation: Given a function $f(x)$, there are many ways for writing its derivative:

$$f'(x) \qquad f' \qquad \frac{df}{dx} \qquad \frac{d}{dx}f \qquad \frac{d}{dx}f(x) \qquad Df(x) \qquad Df$$

So the Chain Rule can be rewritten as: $\dfrac{dz}{dx} = \dfrac{dz}{dy} \cdot \dfrac{dy}{dx}$
(So on the previous page, we had: $12 = 4 \cdot 3$.)

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3. Find the following derivatives:

(a) $\dfrac{d}{dx}[\sqrt{x^2+1}] =$

(b) $\dfrac{d}{dx}[\ln(x^2+1)] =$

(c) $\dfrac{d\,[\ln(x^2+e^{\sin x})]}{dx} =$

(d) $\dfrac{d\,[e^{\ln x}+(x^3+6)^{-1}]}{dx} =$

4. Let $h(x) = f(g(x))$. Complete the following table.

x	2	7
$f(x)$	-1	0
$g(x)$	7	19
$f'(x)$	1.4	8
$g'(x)$	432	.03
$h(x)$		
$h'(x)$		

ANNOUNCEMENTS

Homework, due Monday, 10/19/98: HH, section 4.4: 3, 7, 13, 21, 35, 37; section 4.5: 13, 17.

Second exam: Thursday 10/29, in Lab.