## Warm-Up

Is there any relationship between the graph of a function and the graph of its derivative?

- If a function is DECREASING on an interval [a,b] does this imply anything about its derivative? (If yes, what?)
- What if the function is INCREASING on the interval [a,b]?
- If a function f is CONSTANT on an interval, then this implies f' =\_\_\_\_\_

## Notation:

The derivative (slope, (instantaneous) rate of change) of a function f is defined as:

$$\frac{d}{dx}f(x) = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

provided that all of these limits exist and are equal. In this case, f is said to be *differentiable* at x.

**Derivative of a Constant Function.** For c a constant, the derivative of f(x) = c equals f'(x) =

**Derivative of a Linear Function.** If f(x) = mx + b, then f'(x) =

**Constant Multiple Rule.** If  $F(x) = c \cdot f(x)$ , then F'(x) = \_\_\_\_\_.

**Proof.**  $F'(x) = \lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x}$ 

**Derivatives of Sums and Differences.** If F(x) = f(x) + g(x), then F'(x) =

**Proof.**  $F'(x) = \lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x}$ 

So if 
$$F(x) = f(x) - g(x)$$
, then  $F'(x) =$ \_\_\_\_\_.

**Derivative of a Power Function.** To calculate the derivative of  $x^2$  (using the righthanded difference quotient) we had to multiply out  $(x + \Delta x)^2$ . In general, to find the derivative of  $x^n$  (for n and integer) we will have to multiply out  $(x + \Delta x)^n$ . Let's look at some examples:

$$(x + \Delta x)^{2} = x^{2} + 2x\Delta x + (\Delta x)^{2}$$

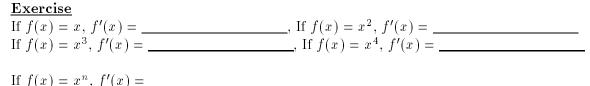
$$(x + \Delta x)^{3} = x^{3} + 3x^{2}\Delta x + 3x(\Delta x)^{2} + (\Delta x)^{3}$$

$$(x + \Delta x)^{4} = x^{4} + 4x^{3}\Delta x + 6x^{2}(\Delta x)^{2} + 4x(\Delta x)^{3} + (\Delta x)^{4}$$

$$\vdots$$

$$(x + \Delta x)^{n} = x^{n} + nx^{n-1}\Delta x + \underbrace{\cdots}_{\text{Terms involving } (\Delta x)^{2} \text{ and higher powers of } \Delta x}$$

Now to find the derivative of  $f(x) = x^n$ :  $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} =$ 



## ANNOUNCEMENTS

The Functions Gateway will be given in Lab #4 tomorrow. Assignment: Due on Friday October 9. CiC page 133-134, DO 2, 5, 6 and 9.