Class 15

Wednesday October 7

Computing Derivatives

Warm-Up

Is there any relationship between the graph of a function and the graph of its derivative?

- If a function is DECREASING on an interval [a,b] does this imply anything about its derivative? (If yes, what?)
- What if the function is INCREASING on the interval [a,b]?
- If a function f is CONSTANT on an interval, then this implies $f' =$ ____________________

Notation:

The *derivative (slope, (instantaneous) rate of change)* of a function f is defined as:

$$\begin{aligned}\frac{d}{dx}f(x) = f'(x) &= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}\end{aligned}$$

provided that all of these limits exist and are equal. In this case, f i s said to be *differentiable at* x.

Derivative of a Constant Function. For c a constant, the derivative of $f(x) = c$ equals

$f'(x) =$

Derivative of a Linear Function. If $f(x) = mx + b$, then $f'(x) =$ ____________________.

Constant Multiple Rule. If $F(x) = c \cdot f(x)$, then $F'(x) =$ ________________.

Proof. $F'(x) = \lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x}$

Derivatives of Sums and Differences. If $F(x) = f(x) + g(x)$, then $F'(x) =$ ____________________.

Proof. $F'(x) = \lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x}$

So if $F(x) = f(x) - g(x)$, then $F'(x) =$ _________________.

Derivative of a Power Function. To calculate the derivative of x^2 (using the righthanded difference quotient) we had to multiply out $(x + \Delta x)^2$. In general, to find the derivative of x^n (for n and integer) we will have to multiply out $(x + \Delta x)^n$. Let's look at some examples:

$$
\begin{aligned}
(x + \Delta x)^2 &= x^2 + 2x\Delta x + (\Delta x)^2 \\
(x + \Delta x)^3 &= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 \\
(x + \Delta x)^4 &= x^4 + 4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 \\
&\vdots \\
(x + \Delta x)^n &= x^n + nx^{n-1}\Delta x + \underbrace{\cdots\cdots\cdots\cdots\cdots\cdots + (\Delta x)^n}_{\text{Terms involving } (\Delta x)^2 \text{ and higher powers of } \Delta x}
\end{aligned}
$$

Now to find the derivative of $f(x) = x^n$:

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} =$

__Exercise__

If $f(x) = x$, $f'(x) =$ ____________________, If $f(x) = x^2$, $f'(x) =$ ____________________

If $f(x) = x^3$, $f'(x) =$ ____________________, If $f(x) = x^4$, $f'(x) =$ ____________________

If $f(x) = x^n$, $f'(x) =$ ____________________

ANNOUNCEMENTS

The Functions Gateway will be given in **Lab #4** tomorrow.

Assignment: Due on Friday October 9. *CiC* **page 133–134, DO 2, 5, 6 and 9.**