Math 110 | BASIC CALCULUS I | Fall 1998

Class 13

Friday 10/02/98

The DERIVATIVE.

Definition: The slope of a function $f(x)$ at a point $x = a$ is also called the derivative of $f(x)$ at $x = a$.

Example: Let $f(x) = x^2 - 1$.

1. $f(\sqrt{2}) = ?$

2. Use the table below to estimate the derivative of $f(x) = x^2 - 1$ at $x = \sqrt{2}$.

x	$f(x)$	$x - \sqrt{2}$	$f(x) - f(\sqrt{2})$	estimate for derivative
2				
1.5				
1.45				
1.415				
1.414215				

So, the SLOPE (or the DERIVATIVE) of $f(x) = x^2 - 1$ at $x = \sqrt{2}$ is EXACTLY

The mathematical way of abbreviating this long sentence is:

3. There is also a third name for *slope, derivative:* ________________________________.

So we denote the derivative of f at $x = \sqrt{2}$ by __________.

Note:

$\dfrac{f(x) - f(\sqrt{2})}{x - \sqrt{2}}$ *is called the* __________ *rate of change of f* **over the interval** ____________,

while

$\lim_{x \to \sqrt{2}} \dfrac{f(x) - f(\sqrt{2})}{x - \sqrt{2}}$ **is called the** ________________ **rate of change of f at** ____________.

Finding the EXACT slope (derivative) of $f(x) = x^2 - 1$ at $x = \sqrt{2}$.

Step 1. Simplify the difference quotient.

$$\frac{f(x) - f(\sqrt{2})}{x - \sqrt{2}} =$$

Step 2. "Take the limit". What happens to your answer in Step 1 as x gets closer and closer to $\sqrt{2}$?

We write: $\lim_{x \to \sqrt{2}} x + \sqrt{2} =$

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4. Find the slope of the parabola $f(x) = 3x^2$ at $x = 1$.
Step 1. Simplify.

Step 2. Take limit.

5. Sketch a graph of the parabola $f(x) = 3x^2$. (Choose a *small scale on your y-axis.)*
(a) On your graph, draw the tangent line at $x = 1$.
(b) On the same graph, draw a line whose slope is "represented" by $[f(1) - f(.5)]/[1 - .5]$.
(c) Without doing any computations, can you tell which is larger: the average rate of change of f on the interval $[.5, 1]$, or the instantaneous rate of change of f at $x = 1$? Why?

ANNOUNCEMENTS

Homework, due Monday, 10/05/98: HH, section 2.2 : 3, 4, 8, 9, 13, 19.
Functions Gateway exam: Thursday 10/08, in Lab.