Math 110 | BASIC CALCULUS I | Fall 1998

Class 5

Euler's Method

Estimating the coffee's temperature more ACCURATELY

Recall: As a hot cup of coffee cools down, it cools down more and more slowly.

Let: C = coffee's temp., C' = rate of change of C.

Suppose when $C = 180$ °F , $C' = -9$ °F per minute.
IF C' was constant, then after two minutes the coffee's temperature would be EXACTLY

$C =$ ____________________

But C' isn't constant.
So after two minutes C will NOT be exactly 162 °F .
Q: Do you expect the true answer to be < 162 or > 162? Why?

<u>Euler's Method</u>: **Use many small time intervals.**

Example:
Recall the rate of change equation: $C' =$ ____________.
Assume $k = -.082$, and $A = 70$ °F .
(a) Find C when $t = 30$ seconds.

(b) Find C' when $t = 30$ seconds.

(c) Repeat for another 30 seconds (i.e., use parts (a) and (b) to find C when $t = 1$ minute).

(d) Find C' when $t = 1$ minute.

(e) Keep repeating until your reach 2 minutes:
find C when $t = 1.5$ minutes;

find C' when $t = 1.5$ minutes;

find C when $t = 2$ minutes.

(f) How could we get an even more accurate estimate for C at $t = 2$ minutes?

Distance, Velocity

1. Suppose a car is travelling at a CONSTANT speed of 80 mi/hr.
(a) How far will it travel in half an hour?

(b) How far will it travel in 20 seconds?

2. Suppose a car is travelling at 75 mi/hr. The driver applies the brakes. The speed of the car is given by $V(t) = 75 - 3t^2$, where time t is measured in seconds elapsed since the brakes were applied, and V is in mi/hr.
(a) What is the speed of the car when $t = 0$?
When $t = 2.5$ seconds?

(b) How long does it take for the car to come to a stop?

(c) Use Euler's Method with one-secont time intervals ($\Delta t = 1$) to estimate how far the car travels before coming to a stop.

(d) If we used half-second time intervals ($\Delta t = 0.5$) instead, would you expect to get a larger, smaller, or the same answer? Why?

Differential Equations (AKA Rate of Change Equations)

1. Suppose y changes with time t according to the equation $y' = 1 + \sqrt{y}$.
(a) What is the rate of change of y when $y = 1.8$?
(b) Suppose when $t = 0$, $y = 1.8$. Use Euler's Method with $\Delta t = .25$ to find $y(1)$.