Class 4

Modelling and Newton's Law of Cooling

Modelling the growth of money.

Suppose Nations Bank pays a 5% return annually on funds deposited into a savings account. If your account contains $ 300 at the beginning of the year, by how much has your account increased at the beginning of the next year?

If M_0 represents the amount of money initially deposited into a savings account, by how much will M_0 increase after a year? What is the total sum of money in the account after a year?
$M(1) =$

After two years? $M(2) =$

After t years?

Let M' represent the rate at which money is increasing (in units of dollars per year). The equation:

$$M' =$$

is a *model* representing how your money grows in the bank.

For this model we were able to come up with a function $M(t)$ which tells us how much money we will have at some point t in the future, if we know the rate $M'(t)$ at which money increases, and the amount of money M_0 we start with.
Write down this function $M(t) =$

Newton's Law of Cooling **states that the rate of cooling is proportional to the difference between the object's temperature and the ambient temperature**. Let C denote the temperature of the coffee (in °F) and let C' be the rate at which it is cooling (in °F per minute.) Let the temperature of the room (ambient temperature) be denoted by A. Newton's Law of Cooling says:

$$C' \propto$$

If the coffee is at a temperature C which is larger than A, will the coffee's temperature go up or down as time goes on? And what does that tell you about C'?

If the coffee is at a temperature C less than the ambient temperature A, will the coffee get warmer or cooler with time? What does that tell you about C'?

Write an equation that relates C' and C. It will contain A, and a constant of proportionality k.
When the coffee is at 180 °F and the ambient temperature is 70 °F , the coffee is cooling at a rate of 9°F per minute. What is k?
Do you *expect* this value to be positive or negative?

At what rate is the coffee's temperature changing after it has cooled down to 135 °F ?

If the temperature of the coffee is initially 180 °F , and it is cooling at 9°F per minute *estimate* its temperature C after 1 minute.

If the temperature of the coffee is initially 180 °F , *estimate* its temperature C after 5 minutes.

If the temperature of the coffee is initially 180 °F , *estimate* its temperature C after 10 and 20 minutes.

If the temperature of the coffee is initially C_0 °F , *estimate* its temperature C after t minutes.

How confident are you of your estimates of the coffee temperature? Do they "make sense"?

Our answers are estimates because the rate at which the coffee cools down **changes** as it's cooling down. In making our estimates, we are assuming that the rate of cooling down is constant over certain time intervals (1 minute, 5 minutes, or 10 minutes.) Unfortunately it is not! **(What time interval would give us the most accurate estimate?)** Which of the following gives a better estimate of the temperature after 10 minutes:

(a) We assume C' is constant for the entire ten minutes. Then $C(10)$ is obtained as we did above.

(b) We assume that C' is constant for the first 5 minutes and calculate $C(5)$ as above. Then recalculate the rate of change, C', based on the new temperature and determine the change in temperature over the next 5 minutes based on the new rate of change. Add this change to the estimate for $C(5)$.

This idea of recalculating the rate of change frequently (every 5 minutes in this case) forms the basis of *Euler's Method* — a technique to approximate solutions to differential equations.