Class 21 HW SOLUTIONS **to selected problems**. Math 110, Fall 98

Was due 10/28/98: HH, section 5.5: 2, 4, 5; CiC, p. 289: 2,.

HH:

2. $S = k_1/x^2 + k_2/(20-x)^2$, so $S' = \dfrac{-2k_1}{x^3} + \dfrac{-2k_2}{(20-x)^3}(-1)$.

Set S' equal to zero:

$$0 = \frac{-2k_1}{x^3} + \frac{2k_2}{(20-x)^3}, \text{ so } \frac{-2k_1}{x^3} = -\frac{-2k_2}{(20-x)^3}.$$

Substitute $7k_2$ for k_1:

$$\frac{-14k_2}{x^3} = -\frac{-2k_2}{(20-x)^3}, \text{ so } \frac{7}{x^3} = -\frac{1}{(20-x)^3}.$$

Now solve for x (some messy algebra), and then use the first derivative test to make sure this critical point is a local minimum. Then, since there are no other critical points, we conclude that in fact it's a global minimum.

4. Let x be the length of the side of the rectangle that is parallel to the long wall, and let y be the length of the other two sides. Then,
$x + y + y = 100$ feet, and area $A = xy$.

We want to maximize A, so use the other equation to eliminate one of the variables:
$x + 2y = 100$, so $x = 100 - 2y$, so $A = (100-2y)y = 100y - 2y^2$.
So, $A' = 100 - 4y$. Set A' equal to zero:
$0 = 100 - 4y$, so $y = 25$. This is a critical point. We MUST check if A has a GLOBAL MAX at $y = 25$:
$A'' = -4$, so A has a LOCAL max at $y = 25$. And because there are no other critical points, there are no other critical points; therefore we have found a global max.
When $y = 25$, we get $x = 100 - 2y = 50$, and $A = xy = 1250$.

5. (a) Let h be the height of the box. Then
$A = 2x^2 + 4xh$, so $h = (A - 2x^2)/4x$.
So, $V = x^2h = x^2[(A-2x^2)/4x] = (Ax - 2x^3)/4$.

(c) $V' = (1/4)(A - 6x^2) = 0$, so $A = 6x^2$, so $x = \sqrt{A/6}$.

$V'' = (1/4)(-12x) = -3\sqrt{A/6} < 0$, so V has a local max at this critical point. As above, since there are no other critical points, this must be a global max.