Basic Calculus I

Class 29

Friday November 16

Single Variable Optimization

The Optimization Problem.

We will deal with some real life problems where it is important to find the maximum and minimum value of some quantity. So in some sense we are trying to optimize the value of some quantity. All the techniques for finding such values make up the field called *optimization*. We shall be able to apply our experience with modelling together with our knowledge of the First Derivative and Second Derivatives Tests to help us in our optimization efforts.

EXAMPLE

[1]. Show that $x > 2\ln(x)$ for all $x > 0$ *(HINT: find the minimum value of* $f(x) = x - 2\ln(x)$*.)* Then use this result to show that $e^x > x^2$ for all positive x.

Exercise

[2]. Show that the rectangle of fixed perimeter P whose area is a maximum is a square.

GROUPWORK

[3]. *Anton, Bivens & Davis*, **Page 313, Example 4**. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

[4]. *Anton, Bivens & Davis*, **Page 318, Homework 1** Find a number in the closed interval $[\frac{1}{2}, \frac{3}{2}]$ such that the sum of the number and its reciprocal is as small as possible.

General method for solving Optimization problems

1. Draw a picture and assign appropriate variables.
2. Write a formula for the quantity to be maximized or minimized.
3. Use the conditions of the problem to eliminate one variable.
4. Determine all of the critical points and intervals the function is increasing and decreasing $(\uparrow, \downarrow)$.
5. Obtain the maximum or minimum, whichever is desired.