Basic Calculus I

Class 28

Wednesday November 14

Global or Absolute Extrema

DEFINITION: global extremum or absolute extremum Given an interval I in the domain of $f(x)$ which contains x_0.

(a) A function f is said to have a **global maximum** or **absolute maximum** at x_0 on the interval I if $f(x_0) \geq f(x)$ for all x in the interval I.

(b) A function f is said to have a **global minimum** or **absolute minimum** at x_0 on the interval I if $f(x_0) \leq f(x)$ for all x in the interval I.

If f has an absolute (global) maximum or absolute (global) minimum at x_0 then f is said to have a **(global) absolute extremum** at x_0.

What kinds of continuous functions have absolute extrema on what kind of intervals?

EXAMPLE

Let's draw some pictures of scenarios. First, let's consider infinitely large intervals (which by definition are open). Remember f is everywhere continuous.

(a) f has one absolute maximum but no absolute minimum on $(-\infty, \infty)$

(b) f has no absolute maximum but one absolute minimum on $(-\infty, \infty)$

(c) f has one absolute maximum and one absolute minimum on $(-\infty, \infty)$

(d) f has no absolute maximum and no absolute minimum on $(-\infty, \infty)$

GroupWork

Now let's consider finite closed intervals. Suppose we have the closed interval $a \leq x \leq b$, sometimes denoted $[a, b]$. Draw a picture of a f which is continuous on $[a, b]$ so that

(a) f has one absolute maximum but no absolute minimum on $[a, b]$

(b) f has no absolute maximum but one absolute minimum on $[a, b]$

(c) f has one absolute maximum and one absolute minimum on $[a, b]$

(d) f has no absolute maximum and no absolute minimum on $[a, b]$

The following theorem explains these results. It is very famous, and it is known as the Extreme Value Theorem.

THEOREM: Extreme Value Theorem

If a function f is continuous on a finite closed interval $[a, b]$ then f must have an absolute maximum and an an absolute minimum on $[a, b]$.

ALGORITHM: How to find Local (Relative) Extrema

1. Determine the domain of the function and identify the end points (if any).

2. Find $f'(x)$.

3. Find the critical points of $f(x)$, i.e. all roots of $f'(x) = 0$ in the domain, and where $f'(x)$ does not exist.

4. Use the First Derivative Test to locate any local extrema. If you want to use the Second Derivative Test, you need to find $f''(x)$ and evaluate $f''(x)$ at the critical points to classify the local extrema.

The local (relative) extrema will occur at the end points (if any) and/or the critical points.

Exercise

Find the local extrema of the function $f(x) = xe^{-x}$ on its domain.

ALGORITHM: How to find Global (Absolute) Extrema

1. Determine local extrema $(c, f(c))$, as above.

2. If the domain is a closed interval of the form $[a, b]$, then check the end point values $f(a)$ and $f(b)$ and determine which value among $\{f(c), f(a), f(b)\}$ is the largest - this is the global maximum; the smallest is the global minimum.

3. If you do not have a closed interval for the domain, then the largest local (relative) maximum is the global maximum, and the smallest local (relative) minimum is the global minimum.

The reason why the previous algorithm works is because of the following theorem

THEOREM

IF a function f has an absolute extrema on an open interval (a, b) THEN it must occur at a critical point of f.

EXAMPLE

Find the global extrema of the function $f(x) = xe^{-x}$ on the interval [0,4].

CLICKER QUESTION

The duration of daylight L in minutes (sunrise to sunset) x kilometers north of the equator on May 17 is given by the function $L = f(x)$. What is are the units of measurement of the seond derivative $f''(4000)$?

(a) kilometers/minute.

(b) minutes/kilometer.

(c) kilometer/minute^2.

(d) minutes/kilometer^2

(e) minutes^2/kilometer^2

CLICKER QUESTION

The derivative of a function is negative everywhere on the interval $x = 2$ to $x = 3$. Where on this interval does the function have its maximum value?

(a) At $x = 2$.

(b) At $x = 3$.

(c) Somewhere between $x = 2$ and $x = 3$.

(d) It does not have a maximum since the derivative is never zero.

(e) We can not tell if it has a maximum since we don't know where the second derivative is negative.

CLICKER QUESTION

TRUE or FALSE. If $f''(a) = 0$, then f has an inflection point at a.

(a) TRUE

(b) FALSE

CLICKER QUESTION

TRUE or FALSE. A local maximum of f only occurs at a point where $f'(x) = 0$

(a) TRUE

(b) FALSE

CLICKER QUESTION

TRUE or FALSE. If $x = p$ is NOT a local extrema of f, THEN $x = p$ is NOT a critical point of f.

(a) TRUE

(b) FALSE