Math 110 **BASIC CALCULUS I** Fall 2007

Class 27

Monday November 12

Analyzing Graphical Behavior Of Functions, Part II

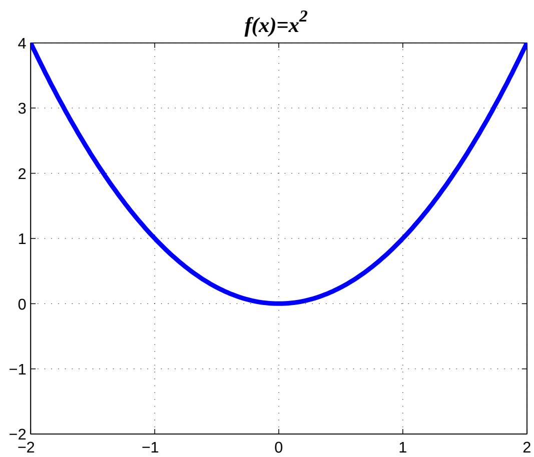
DEFINITION: local extremum or relative extremum

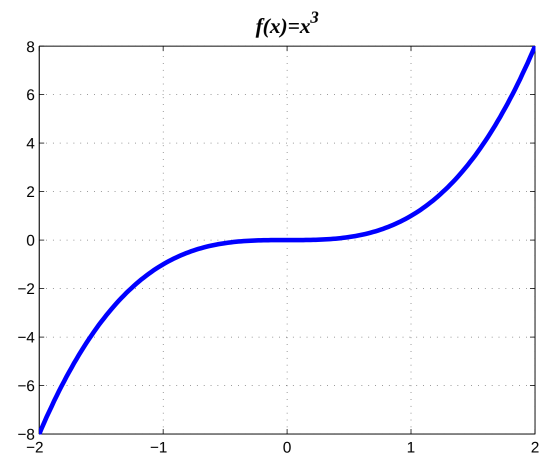
(a) A function f is said to have a **local maximum** or **relative maximum** at x_0 if there is an open interval containing x_0 on which $f(x_0)$ is the largest value, in other words, $f(x_0) \geq f(x)$ for all x in the interval.

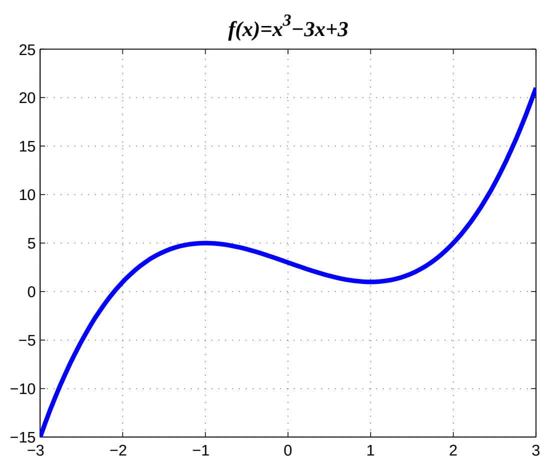
(b) A function f is said to have a **local minimum** or **relative minimum** at x_0 if there is an open interval containing x_0 on which $f(x_0)$ is the smallest value, in other words, $f(x_0) \leq f(x)$ for all x in the interval. If f has a relative(local) maximum or relative(local) minimum at x_0 then f is aid to have an **(local)relative extremum** at x_0. The plural of extremum is extrema.

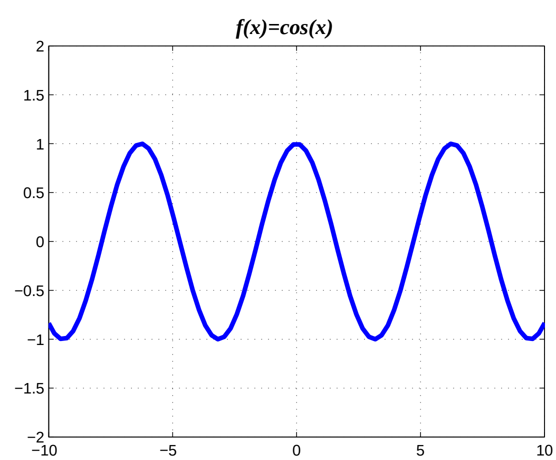
GROUPWORK

Examine the following graphs and identify all the (local)relative extrema.









THEOREM

The First Derivative Test for finding (local) relative extrema

Let $f(x)$ be continuous at a critical point $(c, f(c))$.

If $f^{'}(x)$ is negative to the left of c and positive to the right of c, then $f(x)$ has a (local)**relative minimum** at c.

If $f^{'}(x)$ is positive to the left of c and negative to the right of c, then $f(x)$ has a (local)**relative maximum** at c.

If $f^{'}(x)$ is the same sign to the left of c as it is to the right of c, then $f(x)$ does not have a (local)relative extremum at c.

EXAMPLE

Let's use the first derivative test to find all the local extrema of the functions given on the previous page.

THEOREM

Second Derivative Test for finding (local) relative extrema

Let $(c, f(c))$ be a critical point.

If $f^{''}(c) > 0$ then $(c, f(c))$ is a (local) relative minimum.

If $f^{''}(c) < 0$ then $(c, f(c))$ is a (local) relative maximum.

If $f^{''}(c) = 0$ then $(c, f(c))$ then the test is inconclusive. f may have a (local) relative maximum, a (local) relative minimum or neither at this point. It is possible that $(c, f(c))$ is an inflection point of f.

Exercise

Let's use the Second Derivative test to find all the local extrema of the functions given on the previous page.