Math 110 | **Basic Calculus I** | Fall 2007

Class 23

Monday October 29

Differentiation of Exponential Functions

THEOREM

When $f(x) = b^x$, $f'(x) = b^x \ln(b)$. When $b = e$, $f('x) = f(x) = e^x$ since $\ln(e) = 1$.

GroupWork

Let's understand this result by annotating the following proof of the result. Next to each line of the following steps, write down what mathematical operations have occurred.

$$\begin{aligned} y &= b^x \\ \log_b(y) &= \log_b(b^x) \\ \log_b(y) &= x \\ \frac{1}{y\ln(b)}\frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= y\ln(b) \\ \frac{dy}{dx} &= b^x \ln(b) \end{aligned}$$

NOTE: when $b = e$ (the base of the natural logarithms), $\frac{d}{dx}e^x = e^x$.

EXAMPLE

Let's evaluate the following derivatives

1. Evaluate $\frac{d}{dx}[\pi^x]$.

2. $f(x) = \sin(e^x)$, find $f'(x)$.

3. Evaluate $\frac{d}{dx}[e^{\cos(x)}]$

4. Evaluate $\frac{d}{dx}[2^x x^2]$

CLICKER QUESTION

If $\sqrt{e}$ is approximated by using the tangent line to the graph of $f(x) = e^x$ at $(0, 1)$ and we know $f'(0) = 1$, the approximation is

(a) 0.5

(b) $1 + e^{0.5}$

(c) $1 + 0.5$

(d) $1 + e$

CLICKER QUESTION

The slope of the line tangent to the graph of $x = \sin(y)$ at the point $(0, \pi)$ is

(a) 1

(b) -1

(c) not defined

(d) impossible to be determined

CLICKER QUESTION

If f and g are both everywhere differentiable and $h = f \circ g$, $h'(2)$ equals

(a) $f'(2) \circ g'(2)$

(b) $f'(2)g'(2)$

(c) $f'(g(2))g'(2)$

(d) $f'(g(x))g'(2)$

CLICKER QUESTION

TRUE or FALSE: $\frac{d}{dx} \ln(\pi) = \frac{1}{\pi}$

(a) True.

(b) False.

CLICKER QUESTION

TRUE or FALSE: “There is exactly one function whose derivative equals itself.”

(a) True.

(b) False.

CLICKER QUESTION

TRUE or FALSE: “IF $f(x)$ is an even function, THEN $f'(x)$ is an odd function.”

(a) True.

(b) False.